# APPLIED MECHANICS Reviews

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### GENERAL



### MECHANICS OF SOLIDS



### MECHANICS OF FLUIDS



### HEAT



### COMBINED FIELDS AND MISCELLANEOUS

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// APPLIED MECHANICS REVIEWS

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# DYNAMIC ANELASTIC DEFORMATIONS OF METALS

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## INTRODUCTION

A change in the load acting on a deformable body produces effects throughout the body that are propagated by stress waves. The study of stress-wave propagation in metals is seldom known to be easy even for elastic deformations and is usually extremely difficult for anelastic deformations. In attempting the present survey of the subject of dynamic anelastic deformations of metals, the main purposes are to provide a brief summary of the state of research, to emphasize the essential unity of the subject, and to focus attention on the status of the published literature and on some of the existing gaps in our knowledge. In this short article, a complete coverage is impossible and, indeed, is not regarded as an objective. The selected bibliography provided includes, particularly, the more fundamental papers in the various branches of the subject along with books and review articles dealing collectively with these branches. Much work in the subject has been done at government establishments in various countries, and although some of this work is not generally available, it is, nevertheless, a reasonable assumption that the most important basic research is, in fact, being openly published. The content of this article is focused principally upon research concerning nonlinear stress-wave propagation in metals and this will be found to be a recurrent theme throughout. Thus, highly-specialized topics (such as thermal effects, fracture, and crack propagation) and experimental techniques (such as high-speed photography) will not be discussed. Also, for the sake of brevity, the omission of certain other topics (such as technological metal-forming processes) has been necessary. The bibliography given should be consulted for references to detailed aspects of work in the subject.[1] In detailed references, emphasis is placed upon American and British work but without, it is hoped, any prejudice to Russian work, to which only general references are often given; this procedure has been dictated by the present writer's greater familiarity with the former work rather than with the latter and also by the need for conciseness.[2]

It may be noted here that the development of mathematical-physical theories for solids and liquids has generally proved to be very much more difficult than that for gases, although of course much depends upon the particular physical properties being investigated; essentially, this circumstance is due to the very different situations of atomic or molecular structural order that are involved in the three physical states. The subject of dynamic anelastic deformations of metals is very extensive and it is perhaps usefully regarded as being bordered at opposite extremes by the subjects of the dynamics of elastic solids and of compressible fluids. However, the progress in our knowledge of the present subject differs markedly from that of the others and as yet there are only available rather incomplete and scattered treatments, there still being serious physical and mathematical deficiencies. Thus, much more research will be necessary in order to approach more closely the ultimate objective of reasonably complete investigations, at least for those metals (particularly steels and the light alloys, which themselves embrace a very wide range of mechanical behaviour) that are of most practical importance. Here, of course, concern is primarily with metals in their form of normal practical usage, i.e. often as alloys and invariably as polycrystalline aggregates.

The mechanical behaviour of metals exhibits great variety and often depends not only upon the detailed metallurgical structure but also upon the physical conditions imposed. The understanding and control of the deformations and the stress distributions produced in structures made of metal is a matter of obvious importance, and particular interest accrues to dynamic anelastic situations. Although it is of historical interest to know the origins of work in the subject—which occurred partly in connection with the science of ballistics—it is much more important to observe that by far the major part of the research done has been confined to the last two decades. The development of improved experimental techniques and the occurrence of strong practical motivations have provided the main stimuli for vigorous research. The subject has attracted many workers with widely-varying practical and theoretical interests falling mainly within the fields of mechanics, metal physics, and engineering. Important branches of the subject are pivoted around special types of problems (often in which some simplifying features, e.g. those due to geometrical symmetry, reduce the number of independent and dependent variables involved[3]), and in con-

[1] The considerable extent of the literature may be gauged from the references provided, for example, in the texts by Rinehart and Pearson (6), Cristescu (8), and Goldsmith (11).

[2] The texts by Cristescu (8) and Shestopalov (9) include accounts of Russian work.

[3] It is often found advantageous to give attention to so-called similarity problems in which the number of independent variables is reduced through the introduction of suitable ratios of powers of the original ones. Examples of such problems are given later, and generally the solutions involve mathematical singularities.

sequence the recognition of the subject as a coherent whole often tends to become obscured.

In the mechanics of solids, constitutive equations[4] occupy a central position. In a sense, these equations may be said to constitute a point of arrival for experimental studies and a point of departure for theoretical ones. However, such an ideal meeting-point is often likely to be somewhat spread out due to the difficulty (in all but the simplest cases) of sufficiently isolating those purely mechanical properties of solids whose recognition, elucidation, and experimental measurement are necessary for the construction of constitutive equations that are sufficiently realistic for physical or engineering purposes over the requisite ranges of conditions. The difficulty just mentioned arises particularly in connection with mechanical tests made under dynamic conditions. Thus, the separation of inertia forces, associated either with moving parts of the test apparatus or with the test specimen itself, from the internal forces which arise from and are characteristic of the purely mechanical properties of the solid material is never simple and sometimes impossible to achieve. The response of measuring gauges (mechanical, electrical, or optical) under load introduces further uncertainties, and methods of measurement which do not cause interference with the mechanical properties being investigated are often difficult to find. Another kind of difficulty is that associated with the presence of free surfaces of the test specimen which causes departure from simple stress-strain conditions, e.g. uni-axial stress or strain. Whenever the complete isolation of the mechanical properties is impossible to achieve (at least, to within some acceptable degree of tolerance), then theoretical work is necessary either for the correct interpretation of the experimental data or for the suggestion of different and more suitable mechanical tests. Thus, a subtle combination of both experimental and theoretical work, in which metal physics studies can well be relevant, may be necessary for the construction of appropriate constitutive equations. The implied viewpoint is a vital one in research in the mechanics of solids and no substantial continued progress is likely to be made in the absence of its explicit recognition. It is appropriate to remark upon two additional matters. First, there may be advantages in working with the actual experimental data, rather than with highly simplified models of mechanical behaviour involving a few freely assignable parameters, when deriving detailed quantitative results in the practical application of theories. Second, experimental studies of a given problem (for which, in principle, a theoretical treatment is possible) may be preferable in practice to theoretical studies if these are complicated to handle (this being the case even with certain quite simply-posed problems in the classical dynamical theory of linear elasticity of solids).

Many of the practical situations (particularly those of terminal ballistics) encompassed within the present subject involve the unsteady motion of solid material that takes place over a combination of fundamentally different physical regimes. This means that often a sequence of sets of constitutive equations is required for a sufficient description of the physical behaviour. In these circumstances, a rather severe compromise between physical realities and mathematical complexities is to be expected in theoretical studies, and accordingly only a limited degree of precision is to be envisaged.

In theoretical studies of situations to be described by continuum mechanics, as here, there is necessarily much concern with boundary-value problems of partial differential equations, usually nonlinear or quasi-linear. As the equations involved in the present context often describe phenomena of progressive wave motion, hyperbolic equations are generally expected to occur (although, in certain special cases of physical oversimplification, parabolic equations may occur instead). The natural and most convenient method of solution of these boundary-value problems is then often that based, at least in part, upon the characteristics theory of hyperbolic quasi-linear partial differential equations in two (or more) independent variables. This procedure is relatively straightforward for two-variable problems when it readily yields expressions for wave velocities (often dependent upon the history of the deformation), but it is much less straightforward for three-variable problems. Although the equations admit continuous-wave solutions, it sometimes happens that these build up into discontinuous shock-wave solutions. This circumstance normally results from particular features of the constitutive equations involved and it can arise at widely differing levels of stress amplitude. In physical terms, the situation corresponds to the higher-amplitude stress components (in a pulse that is being propagated) travelling faster than the lower-amplitude ones: thus, the stress pulse undergoes distortion, the leading part becoming progressively steeper and, ultimately, almost vertical. The continuous-wave solution breaks down at this nearly vertical front, and, exactly as occurs in the corresponding situation of compressible fluid dynamics, the front propagates as a shock, i.e. as a very short pulse across which physical quantities, such as stress, strain, and particle velocity, change continuously but very rapidly. Such a narrow shock wave may be quite well approximated for certain mathematical purposes by a discontinuous shock front governed by the Rankine-Hugoniot (finite) equations of conservation of mass, momentum, and energy, but of course the actual shock-wave thickness is determined by dissipative effects such as those associated with rate-of-strain and thermal conduction. An important point to notice is that virtually discontinuous disturbances may result naturally (if only temporarily) from continuous ones. A single system of continuous waves is generated only under rather restricted circumstances, and rather more often considerable complexities arise instead due to the formation of a number of different types of wave systems, undergoing interactions, possibly reflections, and other temporal and spatial changes. The features of the changing wave-pattern may not be too apparent at the outset for any particular case and a high degree of accuracy is normally required with the numerical work involved in their determination. As a result, it is not always easy (or, indeed, possible) to reach general conclusions concerning the features of the stress-wave propagation and separate cases are often likely best to be treated on their own merits.

The main content of this article is presented through a broad partition of the subject into three principal divisions, according as the deformations envisaged occur under conditions of low-amplitude stress (as with low-speed impact or loading conditions), with either independence of effects or dependence upon effects due to rate-of-strain, or under conditions of high-amplitude stress (as with high-speed impact or explosion conditions). In the last case, a subdivision may be made according as whether or not the quasi-fluid mechanical behaviour (i.e. the normal cohesive strength of the solid material is playing little or no part) is sensibly independent of nonlinear compressibility effects, resulting mainly from the variation of the bulk compressibility modulus with the stress amplitude. It is implied that the high-amplitude stress conditions involved are of a compressive nature (not of a tensile nature, this situation being altogether precluded by the onset of fracture) and this is to be understood throughout the sequel. It should be remembered that the above partition is more convenient than precise because, as previously indicated, it is quite possible for widely varying physical conditions to be involved in a given situation.

It is appropriate now to refer briefly to the available texts

[4]This term means the mathematical description of physical properties of materials (e.g. the Hookean stress-strain equations describe linear elastic mechanical behaviour).

and survey articles that bear more generally upon the subject as these references furnish good introductions to its various aspects and parts. The text by Kolsky (1) and the survey articles by R. M. Davies (2, 3, and 4) and Kolsky (5) concern broadly the propagation of stress waves in solids from both experimental and theoretical viewpoints. The texts by Rinehart and Pearson (6), Broberg (7), Cristescu (8), Shestopalov (9), Il'yushin and Lenskiĭ (10), and Goldsmith (11) discuss generally anelastic deformations of metals. The general survey articles by Cristescu (12) and Craggs (13) concern plastic waves. The specialized survey article by Abramson, Plass, and Ripperger (14) concerns stress wave propagation in rods and beams and that by Hopkins (15) concerns spherically-symmetric stress-wave propagation. The text by Bridgman (16) contains the classic early work on the compressibility of metals at high pressure and the survey article by Rice, McQueen, and Walsh (17) on the same subject covers work over a greatly extended range of high pressure. Restricted accounts of the present subject are included in the texts by Courant and Friedrichs (18) and Stanyukovich (19) on unsteady fluid dynamics and allied subjects; by Il'yushin (20) on quasi-static plasticity theory; and by Goodier and Hodge (21) on elasticity and plasticity theory and by Hodge (22) on plastic structural theory.

The broad divisions to be adopted of the subject of the dynamic anelastic deformations of metals have already been given. No discussion can be included here of the suggestions that have been made in order to characterize these separate divisions through the introduction of nondimensional physical parameters (see, however, Refs. 23 and 24). The progress in studies within these separate divisions will now be discussed and appropriate references to the detailed work will be given.

## LOW-AMPLITUDE STRESS LEVELS. RATE-OF-STRAIN INDEPENDENT MECHANICAL BEHAVIOUR

In this Section, attention is confined to a discussion of work concerning those circumstances in which there is complete independence in mechanical behaviour of effects due to rate-of-strain and to high pressure. Thus, in theoretical studies, the governing equations (differential or finite) differ only from those of quasi-static plasticity (e.g. as given in Refs. 25 and 26, being based fundamentally upon Prandtl's laws for idealized mechanical behaviour) by the inclusion of inertia terms.

Exactly as with elasticity studies, anelasticity studies falling within the scope of the present Section may be placed in one of two distinct categories. Thus, there are general treatments of situations in which attention is given to actual stresses and strain-rates, developed particularly under various restrictions primarily dictated by geometry (e.g. under plane, spherical, and cylindrical symmetry-conditions). On the other hand, there are approximate, so-called engineering, treatments of situations in which attention is given to associated generalized (including average) stresses and strain-rates (e.g. stress moments and middle-surface curvature-rates in plates), developed particularly for engineering structural elements (e.g. bars, strings, beams, slabs, plates, and shells[5]) when more precise treatments are scarcely practical or, indeed, necessary (see Ref. 27). It is convenient to discuss separately studies in these two categories, beginning with the engineering treatments.

### ENGINEERING TREATMENTS

*Bars and Wires*—The simplest possible situation concerns the axial deformation produced in a long thin uniform bar or wire by tensile or compressive forces applied at one end. The effect of a nonlinear stress-strain relation (taken, for simplicity, mainly as a bilinear one) on the propagation of axial stress was first given attention by Donnell (28) who found that a continual change in shape of a stress pulse would occur. No further interest then seems to have been shown in the problem until World War II when more detailed studies were developed independently by Taylor (29, 30, and 31), by von Kármán and Duwez (32; see also Ref. 33) and White and Griffis (Refs. 34, 35, and 36), and by Rakhmatulin and Shapiro (Refs. 37, 38, and 39) and Stanyukovich (19).

The basic problem first solved in detail by von Kármán is described as follows. The end of an infinitely-long uniform bar is suddenly constrained to move with velocity $V$, maintained constant for all later times, so that tensile strains occur. The following are the main assumptions made in the theoretical treatment of this problem: transverse inertia effects are negligible; the stress-strain curve is invariable, rate-of-strain sensitivity of the material being negligible; and this curve is concave towards the strain axis, strain being a single-valued function of stress.[6] Of these assumptions, the last excludes the formation of shock waves[7] but the others necessarily involve a varying degree of approximation, depending upon the circumstances of any particular case, and this is discussed later. The analysis is most simply made in terms of nominal or engineering stress $\sigma$ (i.e. force per unit initial cross-sectional area) and strain $\varepsilon$ (i.e. change in length per unit initial length, not necessarily small), it being supposed that the static stress-strain relation $\sigma = f(\varepsilon)$ is known. Then, in the Lagrangian co-ordinate system, the wave velocity $c$ and particle velocity $v$ are given by

$$c = -(T/\rho_0)^{1/2},\ v = \int_0^{\varepsilon} (T/\rho_0)^{1/2}\, d\varepsilon,$$

where $T = d\sigma/d\varepsilon$ is the tangent modulus and $\rho_0$ is the initial density. The determination of the plastic wave propagation is, in fact, particularly simple because the specification of the problem introduces no fundamental length, and there exists a similarity solution, being such that quantities are essentially expressed functionally in terms of a single independent variable $\xi = x/t$ (where $x$ is the initial distance of an arbitrary cross section from the impacted end and $t$ is time after the beginning of the impact). In more detail, the principal results are as follows. The front of the disturbance travels at the velocity of longitudinal elastic waves, and immediately at this elastic wave front there is a discontinuity in strain corresponding to that for initial yield, say $\varepsilon_y$.[8] Behind the elastic wave front, there is a plastic disturbance (which corresponds to a continuous wave), the velocity of wave propagation varying with the strain and gradually falling-off as the strain increases beyond $\varepsilon_y$ up to some value $\varepsilon_1$ (a function of $V$ and the stress-strain relation), obtaining at the so-called plastic wave front. Thereafter, right up to the struck end of the bar, the strain has the uniform value $\varepsilon_1$. If the strain $\varepsilon$ is plotted as a function of $\xi$, then the curve obtained is the same for all time as shown in Figure 1.

A most remarkable feature of the solution is the prediction of a flat plateau of strain extending over a region that steadily grows outwards from the impacted end of the bar. In practice, fracture (following necking) will be produced if $\varepsilon_1$ reaches some critical value $\varepsilon_{cr.}$, dependent upon the ultimate strength of the material, and therefore there is a corresponding critical velocity $V_{cr.}$ in excess of which fracture at the end of the bar

[5]The general engineering meaning of such terms is intended here.

[6]A fascimile of the letter, written by von Kármán to Duwez, in which the essentials of the theory were first given is exhibited in Ref. 33.

[7]Strictly speaking, a purely elastic stress pulse eventually results in a very weak shock.

[8]In 1807, T. Young (see Ref. 32) has noted that some permanent strain would occur when the impact velocity exceeded a certain critical value.

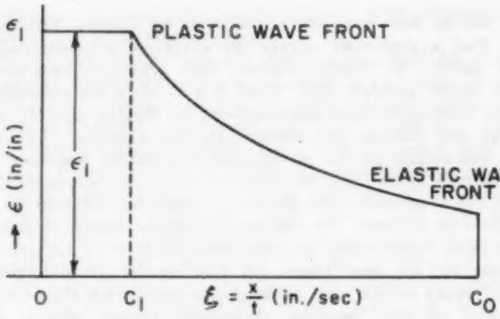


**Fig. 1.** Strain distribution produced in a wire by longitudinal impact at its end.
Schematic representation of $\varepsilon$ versus $\xi = x/t$.
(From von Kármán and Duwez, Fig. 1, p. 988, Ref. 32.)

occurs almost instantaneously and then comparatively little wave-propagation occurs.

During 1940–43, von Kármán in collaboration with H. F. Bohnenblust, J. V. Charyk, D. S. Clark, P. E. Duwez, D. H. Hyers, and D. S. Wood developed the basic theory (using Lagrangian co-ordinates) of longitudinal plastic waves in semi-infinite and finite bars and wires and undertook supporting experimental work. This work (like Taylor's and also White and Griffis', discussed later) was described in official reports but was only published much later (see Refs. 32 and 33). For the particular problem described above, it was intended that the experimental work should provide a check upon the principal theoretical results: viz. the existence of a strain plateau of definite amplitude $\varepsilon_1$; the relation between $\varepsilon_1$ and the impact velocity $V$; and the variation of strain between the elastic wave front and the strain plateau. In general, the first two results appeared to be fairly well substantiated but the most significant and systematic discrepancies found, occurring mainly between measured and predicted strain variations, were ascribed to rate-of-strain sensitivity. Mention should be made also of work by White and Griffis (Refs. 34, 35, and 36) which included attention to plastic shock-wave propagation, this occurring, for example, when the engineering stress-strain curve is concave towards the stress axis (e.g. as with nickel-chrome steel and copper over certain ranges of compressive strain).

Taylor (29) in 1940 developed the basic theory (using Eulerian co-ordinates) of one-dimensional finite-amplitude plastic wave propagation and used a version of this theory to discuss the propagation of earth waves from an explosion. He noted that the governing equations were formally identical with those for the propagation of one-dimensional finite-amplitude waves in a compressible perfect fluid (the pressure being a function only of the density), although the correspondence between the two situations exists only for loading waves as solids exhibit hysteresis in the plastic region. For this latter situation, Riemann and also Earnshaw (see Ref. 40) in now classic work had given the general solution which established that a unidirectional disturbance was possible. Taylor (30) noted later (in 1942) that his own plastic wave theory (which he had also applied to discuss the plastic wave in a wire struck by a bullet) was similar to that of von Kármán,* and he now formally presented his earlier theory of longitudinal plastic wave propagation along a wire and also commented upon von Kármán and Duwez's experimental results. In the Eulerian co-ordinate system (but otherwise with the same notation given above in the Lagrangian treatment), the wave velocity $c$ and particle velocity $v$ are given by

$$c = v - (1+\varepsilon)(T/\rho_0)^{1/2}, \quad v = \int_0^{\varepsilon} (T/\rho_0)^{1/2}\, d\varepsilon.$$

Rakhmatulin and Shapiro (see Refs. 37, 38, and 39) also developed the basic theory (using Lagrangian co-ordinates) of plastic wave propagation along bars. This theoretical work was extended by other Russian workers to cover a wide variety of situations and supporting experimental work was also undertaken (for references to this theoretical and experimental work, see Refs. 8, 9, 10, and 12). Stanyukovich (19) also discussed problems of plastic wave propagation independently of the work by Rakhmatulin.

The pioneer work of Taylor, von Kármán, and Rakhmatulin, which in a sense may be regarded as a development of Donnell's much earlier work although this was not apparently widely known at the time, marks the beginning of the intensive study of the propagation of plastic waves in solids. In particular, this work has led to numerous experimental and theoretical investigations being made of plastic wave propagation along bars and wires. In fact, due to its supposed simplicity (although, as will be seen later, this is more apparent than real), this area of the present subject has received the greatest amount of individual attention.

It is convenient now to make certain general remarks concerning one-dimensional plastic wave propagation. A single system of continuous waves is generated only under restricted circumstances, i.e. when there is increasing (or constant) applied load acting at the end of a semi-infinite bar, the tension or compression engineering stress-strain curve being concave towards the strain axis, shock waves not then occurring. If, after a time, the applied load decreases (or is entirely removed), unloading elastic waves are propagated forwards in the rear of the original loading elastic and plastic waves; and, if the bar is finite in extent, then waves will ultimately be reflected back at the distal end. The occurrence of shock waves is generally to be expected (if not immediately, then later) when the stress-strain curve is concave towards the stress axis, for then higher-amplitude stresses are propagated more rapidly than lower-amplitude ones (see previous discussion in the Introduction). Thus, it is apparent that considerable complexities may arise due to the presence of a number of different wave-systems, undergoing interactions, reflections, and other temporal and spatial changes. In such circumstances, it now becomes necessary to follow carefully the strain path associated with any given cross-section of the bar, the local wave velocity being dependent upon the history of the deformation. In general, the natural and most convenient method of solution of the boundary-value problems that arise is based, at least in part, upon the theory of characteristics of hyperbolic partial differential equations in two independent variables (see, for example, Refs. 18 and 41). However, as indicated, the details of the procedures required, which include integration along characteristics, may become rather complicated, particularly if shock waves are involved.

The most important conclusion from all the extensive work on longitudinal plastic wave propagation along bars and wires is that the simple, rate-of-strain independent, uni-axial theory is reasonably satisfactory only for those materials without a well-defined yield-point and an undue rate-of-strain sensitivity (mild steel, of course, violates both these conditions). The importance of characteristics techniques and the complexities occurring with multiple-wave systems are to be noted.

*Strings*—Transverse and longitudinal plastic wave propagation in strings was discussed by Rakhmatulin (42) and other Russian workers, and a summary of this work is given by Cristescu (8 and 12). Other work is due to Craggs (43) and Karunes and Onat (44).

*The essential equivalence of the two theories was established by H. F. Bohnenblust and by others (see Refs. 30 and 4).

*Beams*—Transverse plastic deformation in beams was discussed first by Duwez, Clark, and Bohnenblust (45). The governing equations, being of parabolic type, do not admit solutions corresponding to progressive wave motion. The only solution known, applying for elastic-plastic material, is for the similarity problem of a long beam subjected to uniform transverse velocity impact at a fixed cross-section. As established much earlier by Boussinesq (in 1885) for the corresponding problem of an elastic beam, quantities are then essentially expressed functionally in terms of a single independent variable $\eta = x^2/t$ (where $x$ is distance measured along the beam from the place of impact and $t$ is time after the commencement of impact). In marked contrast, much simpler analyses for a wide variety of problems may be derived if certain approximations in mechanical behaviour are made. Thus, Lee and Symonds (46) showed that the concept of plastic hinges, previously used extensively in the solution of static problems of perfectly-plastic, rigid beams (see Ref. 22), could be used also in the solution of dynamic problems. In the simple engineering theory, the bending moment at a plastic hinge is numerically equal to the fully-plastic moment. Both isolated and distributed plastic hinges are possible, and generally the transverse motion of beams is accompanied by the longitudinal movement of plastic hinges. The simplicity of the analyses of problems is essentially due to the fact that the mechanical systems considered now have only a finite number of degrees-of-freedom and rigid-body mechanics is applicable. Very many theoretical studies based upon perfectly-plastic, rigid mechanical behaviour have been made (for references, see Ref. 13), and some experimental studies have also been made (see, for example, Ref. 47).

*Slabs and Membranes*—Plastic wave propagation in membranes has been discussed generally and also with attention to specific problems (for references, see Refs. 12 and 13). In particular, Rakhmatulin (48) has discussed torsional waves and Freiberger (49) has discussed the enlargement of a circular hole; and general analyses of longitudinal and transverse wave propagation are due to Cristescu (8) and Craggs (13).

*Plates*—Studies of dynamic transverse deformations in plates have involved one of two different approaches, applicable according as the conditions permit the plate to be regarded as either thick or thin. Thus, Bakhshiyan (50) and later Kochetkov (51), for visco-plastic and elastic, visco-plastic types of mechanical behaviour respectively, assumed that the dominant mode of deformation was one of transverse shear and discussed the problem of the transverse impact of a rigid circular cylinder with a plate. On the other hand, Hopkins and Prager (52) assumed that bending deformation was the dominant mode (i.e. exactly as with the elastic theory of thin plates) and showed that the concept of plastic-hinge lines was applicable to dynamic problems of perfectly-plastic, rigid plates. This latter approach makes for much simplicity and it has been adopted in the solution of a number of problems usually involving rotational symmetry (for an exception, however, see Ref. 53). On the different basis of deformation theory, Cristescu (54) has recently discussed the propagation of plastic waves in a plate subjected to combined transverse and rotational impact by a rigid circular cylinder.

*Shells*—Studies of dynamic deformations in shells are restricted to cylindrical shells. Torsional deformation has been considered approximately by H. Wolf (see Ref. 13). The more difficult situation of radial deformation has been considered by Hodge (55 and 22), Eason and Shield (56), and Chiapetta and Hodge (57) for perfectly-plastic, rigid material; their analyses all involve approximations to the generalized yield criteria which strictly apply and use is made of the concept of plastic-hinge curves.

### General Treatments

*Plane waves*—Plane wave propagation occurs when the motion is dependent only upon a single rectangular Cartesian co-ordinate and the time; the waves may be longitudinal or transverse. Craggs (13) has given a general discussion and has derived analytical expressions for the wave velocities. An important special case of plane wave propagation is that involving uni-axial strain which occurs when the plane surface of a region of uniform (or infinite) depth, but otherwise of infinite extent, is subjected to a uniform pressure. Under these conditions, although only the axial strain is non-zero, there are both axial and transverse stresses. In a more general context than that of the present Section, Wood (58) and Morland (59) have given attention to uni-axial wave propagation, this bearing certain resemblances to longitudinal wave propagation along bars, discussed above.

*Spherical waves*—Spherical wave propagation occurs when the motion is dependent only upon a single radial (spherical polar) co-ordinate and the time; one principal direction coincides with the radial direction about which there is isotropy. The important feature is that, due to spherical divergence (or convergence), there are changes in both amplitude and shape of stress pulses. Hopkins (15) has summarized work on spherical elastic-plastic wave propagation. As elsewhere, this work has involved the extensive use of characteristics techniques.

*Axisymmetrical waves*—Axisymmetrical wave propagation occurs when the motion is entirely independent of the angular (cylindrical polar) co-ordinate; one principal direction coincides with the angular direction. Cylindrical divergence affects wave propagation in causing changes in amplitude and shape of stress pulses. Generally, the problems of axisymmetrical (and also of cylindrical) wave propagation are fundamentally more difficult than those of spherical wave propagation and much less progress has been achieved. Cristescu (8; see also Ref. 13) has given general analyses of axisymmetrical wave propagation, and the special case of purely torsional wave propagation has been discussed by Rakhmatulin (48).

*Plane-strain deformations*—A somewhat unusual study of dynamic plane-strain deformations has been made by Spencer (60) who included discussion of the disturbance produced in a semi-infinite region of perfectly-plastic, rigid material by the forced piston-like motion of a rigid flat punch.

## LOW-AMPLITUDE STRESS LEVELS. RATE-OF-STRAIN DEPENDENT MECHANICAL BEHAVIOUR

The classic work of J. Hopkinson (61 and 62) and B. Hopkinson (63) provided the first clear demonstration that the mechanical behaviour of metals under dynamic loading conditions could be significantly different from that under static ones. It is now well established that the mechanical behaviour of metals is often dependent upon the rate-of-strain. In this case, the instantaneous strength and the energy absorption achieved for a given strain increase with the rate-of-strain and the magnitudes of these changes vary considerably from one metal to another. The proportional change in strength ranges from being quite large, as for mild steel, to being almost negligible, as for some high tensile steels. In physical terms, the effect is thought to be associated with the times needed for atomic lattice imperfections, such as dislocations, to form and to move across crystal grains and thereby to permit plastic flow to occur (see, for example, Ref. 64); the effect of rate-of-strain is accordingly highly sensitive to the metallurgical structure, and there are often considerable variations in its magnitude, even between different alloys of the same metallic element. The Hopkinsons—father and son—both carried out experiments in which a vertical steel wire, fixed at its upper end, was suddenly stretched at its lower end by the impact of a weight falling from a given height. J. Hopkinson obtained the remarkable result that the least height-of-fall required to cause the wire to break (which usually occurred near the top) was independent of the actual weight used. Although

plastic deformation of the wire certainly occurred in some of his experiments, the results obtained were given an explanation in terms of longitudinal elastic wave propagation. J. Hopkinson concluded (although somewhat prematurely, as remarked upon by Taylor in Ref. 31) that, under the conditions of his experiments, the impact blows were equivalent, not for equal momenta or energies, but for equal velocities. B. Hopkinson repeated his father's experiments, but with improved techniques he was now able to apply impact blows which caused the wire to be strained only elastically. He showed that the dynamic yield strength of the steel wire was nearly twice the static one, thereby substantiating his father's conclusion. For his particular experiments, B. Hopkinson assumed (however, incorrectly) that the maximum stress occurred at the top of the wire after the first wave reflection, but Taylor (31) has shown that this happened after the third wave reflection and thereby reinforced B. Hopkinson's conclusion. B. Hopkinson gave the static yield stress of the wire as 18 tons/in.$^2$ whereas Taylor's analysis of his experimental results showed that a dynamic stress greater than 28 tons/in.$^2$ was supported for $10^{-4}$ sec. without producing permanent deformation.

A very great deal of work has been done in order to devise experimental techniques aimed to provide data on the mechanical properties of metals under dynamic conditions of applied load (see, for example, Refs. 1 and 11). However, this work is often much complicated by the difficulty of isolating sufficiently well the mechanical properties to be measured (see previous discussion in the Introduction). At low rates-of-strain, modifications of standard static mechanical testing machines have led to the design of satisfactory apparatus so that the load actually applied to the test specimen and the uniform strain in it are both accurately measurable quantities. At medium rates-of-strain, the inertia forces in the moving parts of the mechanical system through which load is applied can cause uncertainties in measurements of the load actually applied to the test specimen. At higher rates-of-strain, the inertia of the test specimen itself can cause stress waves with times-of-transit along the test specimen comparable with times-of-increments of applied load. Generally, the difficulties in obtaining and in interpreting experimental data increase with increasing rate-of-strain: they occur at relatively low rates-of-strain with mechanical testing machines but may be delayed or minimized up to higher rates-of-strain through the use of different experimental techniques. Amongst others, Taylor (31), Clark and Duwez (65), Habib (66), Lee and Wolf (67), and Campbell (68) have paid attention to this very important question; some discussion of their work and other more recent work is given below.

One of the earliest systematic investigations of rate-of-strain effects in metals is due to Ludwik (69) who found empirically a logarithmic dependence of yield strength upon rate-of-strain, a result supported (at least, over certain ranges of rates-of-strain) by very many investigations made since that time (see Figure 2). Mechanical testing machines have been developed in which short specimens are subjected to a uniform rate-of-strain and extensive experimental data for rates-of-strain up to the order of $10^3$ sec.$^{-1}$ have been obtained in this way (see, for example, Refs. 11 and 70).

One experimental technique in which the yield-point of the material is not exceeded is due to Taylor (31) and this has been developed by R. M. Davies (71). Measurements are made of the least static load required to indent a steel surface with a hard steel ball and of the least height from which this ball must be dropped in order to produce an indentation on the same surface. Through use of Hertz's theory of impact (which assumes that the stress distributions near the contact area are geometrically similar for both static and

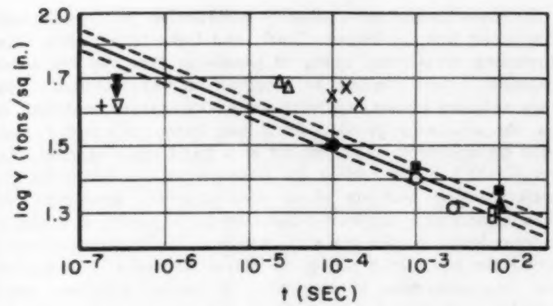


Fig. 2. Dependence of yield strength upon rate-of-strain. Comparison of results for mild steel. Yield strength, $Y$, against time to yield, $t$. The symbols denote experimental results of various investigators.
(From Costello, Fig. 2.8, p. 20, Ref. 121.)

dynamic conditions), it is possible to deduce the ratio of the values of the dynamic and static yield-stresses, supposing that yield occurs at similar positions under both loading conditions. For steels, this ratio was found to decrease from about 2 to 1 as the yield strength increased from that for mild steel to that for high tensile steel. The rate-of-strain under the present dynamic conditions is extremely high, the time-of-contact between the ball and the plate being of the order of $3 \times 10^{-8}$ sec. The results obtained in this way are also of interest in connection with the dynamic hardness-testing of metals, although concern there is more with fully-developed plastic flow (see Ref. 72).

In some other simple techniques of the dynamic testing of materials, the yield-point is passed. Brown and Vincent (73) have devised apparatus in which the load is applied mechanically, a piezo-electric crystal to measure stress being placed between the far end of the plastically-strained test specimen and an anvil, and strain in the specimen being measured optically. Thus, both stress and strain are recorded simultaneously. The results obtained are in agreement with those of B. Hopkinson, and Taylor and R. M. Davies, already discussed, and also of other experimenters. The characteristic feature of another test, devised by Taylor (31) and H. Quinney in order to avoid vibration troubles, is that the applied loads are inertia ones and are quite accurately determinable. In this test, the impact of a lead bullet with an anvil is used to strain plastically twin, parallel, short test specimens whose far ends are attached to a double ballistic pendulum, and values of dynamic yield-strength at various rates-of-strain are deduced from the observations. The maximum rate-of-strain achieved in this test is of the order of $3 \times 10^3$ sec.$^{-1}$, and the corresponding value in Brown and Vincent's test is $6 \times 10^2$ sec.$^{-1}$.

In the above types of experiments devised by Taylor (31) and Brown and Vincent (73), analyses of the data obtained are possible because the conditions are such that either elasticity theory is applicable or there is sensibly uniform plastic strain of the test specimen, again permitting the application of a simple theory.

As already mentioned, at higher rates-of-strain, satisfactory tests require the avoidance of certain immediate difficulties: first, inertia loads in the moving parts of the test apparatus can cause uncertainties in the true values of applied loads (see Ref. 65); and second, inertia of the test specimen itself can cause stress-wave propagation leading to nonuniform conditions (see Ref. 67). The first difficulty may be avoided in various ways: by applying inertia loads, as with a bullet or a long elastic bar (hung as a ballistic pendulum) impacting on the test specimen or with the test specimen itself

impacting on a long elastic pressure-bar; and by applying loads to the test specimen through a long elastic pressure-bar. The applied loads may be readily found. The second difficulty is the more serious. In general, a correct interpretation of the experimental data then requires analysis based upon some particular theory of plastic wave propagation, but this procedure is only valid if the theory used is in fact substantially correct.

Two main approaches to experimental techniques of dynamic testing at higher rates-of-strain have been made. First, there is work concerning tests that involve the firing of cylindrical specimens at a thick armour steel plate (see Refs. 74, 75, and 76), the maximum rate-of-strain being of the order of $2 \times 10^4$ sec.$^{-1}$. A comparison between experimental and theoretical results for a particular case is shown in Figure 3.

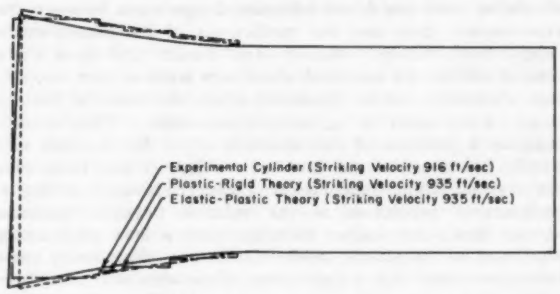


**Fig. 3. Plastic deformation in a steel cylinder striking a rigid target. Comparison of final shape of cylinder, striking velocity 916 ft./sec., with calculated shapes for a striking velocity of 935 ft./sec., obtained from elastic-plastic and plastic-rigid one-dimensional theory.**
**(From Lee and Tupper, Fig. 13, p. 69, Ref. 76.)**

Second, there has been much work involving the use of pressure-bar techniques. The pressure-bar technique (see Ref. 1) was first devised by B. Hopkinson (77) in 1914 in order to investigate the propagation of elastic stress pulses; an electrical version of this technique was developed by R. M. Davies (78) in 1948 and the so-called split pressure-bar was introduced by Kolsky (79) in 1949. The application of these techniques to the dynamic testing of materials at high rates-of-strain, which can involve certain further difficulties associated with lateral inertia and interfacial friction, was first made by Volterra (80) and Kolsky (79), and it is discussed later. The particular difficulties just mentioned occur also in the first approach as well.

Work by J. D. Campbell and his collaborators is of a different kind and has concerned the determination of dynamic tensile stress-strain curves right up to fracture. Thus, Harding, Wood, and Campbell (81) have developed a method of testing for rates-of-strain of the order of $10^3$ sec.$^{-1}$ that involves the use of elastic wave propagation theory in order to interpret the experimental data. Also, work by Austin and Steidel (82) has concerned particularly the high rates-of-strain occurring during the final stages of plastic deformation up to fracture. In their experimental method, test specimens of steel and aluminium alloy are extended directly, by the motion of an explosive projectile, at rates-of-strain of the order of $2 \times 10^4$ sec.$^{-1}$.

The dependence of the mechanical behaviour for metals upon the rate-of-strain is now very well established, and various workers have suggested that the constitutive relation obtaining in the simple tensile test is of the form

$$\sigma = \phi(\varepsilon_p, \dot{\varepsilon}_p) \qquad (*)$$

under loading conditions, where $\sigma$ is nominal tensile stress and $\varepsilon_p$ and $\dot{\varepsilon}_p$ are nominal plastic strain and strain-rate. Thus, a logarithmic form for $\phi$ was proposed by Ludwik (69) and Deutler (83) on empirical grounds and by Prandtl (84) as a prediction of a physical theory of plastic flow (see also Refs. 11 and 70). For clarity, Malvern (85 and 86) has expressed the proposed dynamic constitutive relation in the form

$$\sigma = f(\varepsilon) + a \ln(1 + b\,\dot{\varepsilon}_p),$$

where $\sigma = f(\varepsilon)$ is the static stress-strain relation, $\varepsilon$ is nominal total strain, and $a$ and $b$ are physical parameters. It should be noted that, generally, mechanical behaviour is not too dependent upon the precise range of rate-of-strain, and that for practical purposes it is therefore often convenient to refer simply to the order of loading duration, e.g. second, millisecond, or microsecond. Malvern's relation may be re-written as

$$b\,\dot{\varepsilon}_p = \exp\{(\sigma - f)/a\} - 1,$$

i.e. the plastic strain-rate is simply a function of the difference between the dynamic and static stresses achieved at the same total strain. If, as here, the plastic strain-rate is represented as a function of an over-stress, then, as Lee (87) has remarked, it would be physically more satisfactory, although mathematically a little less simple, to define the over-stress with respect to plastic strain $\varepsilon_p = \varepsilon - \sigma/E$ (where $E$ is Young's modulus) and not to total strain $\varepsilon$. However, the difference involved is negligible in most cases, and, in any event, transformation between the two definitions is easily effected. Malvern (85) has developed a longitudinal stress-wave propagation theory on the basis of the more general dynamic constitutive relation that corresponds to (*) above, viz.

$$E\dot{\varepsilon} = \dot{\sigma} + g(\sigma, \varepsilon),$$

where $g$ is an arbitrary function expressing the rate-of-strain dependency. In this relation, elastic strain is assumed independent of the rate-of-strain and no account is taken of lateral inertia effects (later discussed). Sokolovskiĭ (88 and 89) previously considered a special case of this constitutive relation applying for non-workhardening material. In Malvern's rate-of-strain dependent theory of longitudinal plastic waves, the governing equations form a hyperbolic system that is numerically integrable when the boundary conditions are assigned and when the form of $g$ is chosen. Malvern's constitutive relation has certain immediate interesting implications. First, there is an apparent increase in the value of the initial yield-stress at high rates-of-strain because plastic strain does not occur instantaneously but takes time in which to become appreciable. Second, small increments of strain superimposed upon a static plastic-strain distribution are propagated at the elastic wave velocity and not at the appropriate plastic wave velocity as predicted by the Taylor-von Kármán-Rakhmatulin rate-of-strain independent theory already discussed. These predictions appear to be in broad agreement with certain experimental results, and the second one, particularly, is referred to later in this Section. For purposes of illustration and for simplicity, Malvern took the quasi-linear form $g = k(\sigma - f)$ where $k$ is a constant determining the magnitude of the rate-of-strain effect (rather than taking the exponential form above which involves two physical constants), and he chose a value of $k$ suggested by Habib's (66) experimental data for copper and values of other quantities and a simple form for $f$ representative of hardened aluminium. On this basis, Malvern derived a rate-of-strain dependent solution of the basic problem discussed in the early work on plastic stress waves in bars and wires, viz. wave propagation under conditions of constant velocity impact. It was found that there was now no uniform strain plateau near

the impacted end of the bar and that the maximum strain was increased, as shown in Figure 4. These results (which persist when the impact is not applied instantaneously), he concluded, were opposite to those required to improve agreement between experiment and theory. Although the exponential form for $g$ expressing rate-of-strain dependence might give more satisfactory results, Malvern thought it likely that this form, (and indeed any form different from the rate-of-strain independent one, $g = 0$) must fail to predict the uniform strain plateau apparently observed. However, not all the experimental results have definitely predicted such a plateau: Malvern observed that no plateau was found in certain tests made on lead bars, and Lee (87), in discussion of Malvern's work, indicated that more accurate analysis of the available experimental data showed that this was also the case for copper wires. Lee also discussed the close relation between Malvern's work and other work concerning wave propagation in visco-elastic materials (see also Ref. 90).

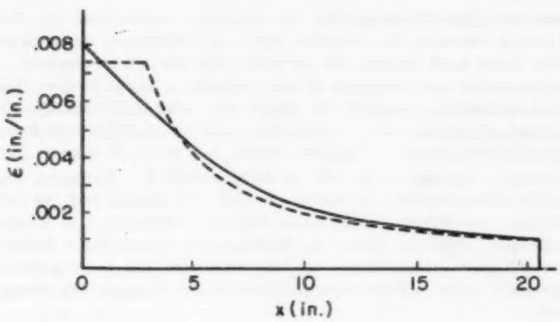


Fig. 4. Strain distribution produced in a wire by longitudinal impact at its end. Comparison between results of rate-of-strain dependent theory (Malvern) and rate-of-strain independent theory (Taylor-von Kármán-Rakhmatulin). Strain distribution at $t = 102.4$ microsec. (broken line is solution neglecting strain-rate effect). (From Malvern, Fig. 6, p. 206, Ref. 85.)

As already discussed, the limitations of the Taylor-von Kármán-Rakhmatulin rate-of-strain independent theory of plastic stress wave propagation along bars were early appreciated, it being concluded that the rate-of-strain sensitivity in the mechanical behaviour of certain metals was not always a negligible effect. Malvern's rate-of-strain dependent theory, although clearly still involving considerable simplifications of true mechanical behaviour, has generally been regarded as effecting a marked improvement in approaching the development of a more realistic theory incorporating rate-of-strain effects. This theory has resulted in much renewed interest in the subject (see Ref. 87), and, in particular, a number of further experimental studies involving pressure-bar techniques has been made, as discussed below.

Bell (91) in tests made on steel bars found that incremental impact loads superimposed upon a static load in excess of the elastic limit were propagated at the elastic wave velocity. Sternglass and Stuart (92) investigated the same situation as that in Bell's work, viz. the propagation of a small loading pulse along a statically preloaded plastic bar; their tests made on copper strips showed, in particular, a similar result. Other workers have found the same result, and a theoretical study allied to Malvern's work concerning rate-of-strain dependent longitudinal wave propagation has been given by Rubin (93). Alter and Curtis (94) considered a related type of experiment in which lead bars were subjected to a first impact load, which plastically preloaded the bar, followed by a second impact load: the front of the second disturbance was found to propagate at the elastic wave velocity. It should be remembered, however, that lead has a low yield-point and is a notoriously time-dependent metal in its mechanical behaviour. Bodner and Kolsky (95), who have also made experimental studies of stress-wave propagation in lead bars, suggest that in this respect the mechanical behaviour of lead may perhaps be regarded as being somewhat more analogous to that of visco-elastic materials (such as plastics and other high polymers) than to that of a more typical metal (at normal temperatures), in which case a visco-elastic relation effect could serve to explain the observed phenomena (see Ref. 87). All the above results show, of course, a real qualitative difference from the predictions of the rate-of-strain independent theory. Bell (96, 97, and 98) in studies of constant velocity impact on annealed aluminium bars has found substantial agreement between the experimental data and the predictions of the rate-of-strain independent theory. Kolsky and Douch (99) have made similar studies for annealed short bars made of pure copper, pure aluminium and an aluminium alloy, the times of loading being of the order of tens-of-microseconds. Their results indicate a pronounced rate-of-strain effect for the first two metals but none for the last one. Also, it was found that the rate-of-strain independent theory, although giving a satisfactory prediction of the relation between maximum plastic strain and impact velocity, gave a less satisfactory prediction of the plastic strain distribution, there being some indication that the experimental data obtained from low-velocity impact tests on copper bars would be more in accord with the predictions of Malvern's rate-of-strain dependent theory. Kolsky and Douch found further that the maximum stresses that could be propagated at the elastic wave velocity along annealed bars of the three metals were in reasonable agreement with the static yield-stresses; this result apparently appears to conflict with the results of Bell (91) and Sternglass and Stuart (92), but it must be remembered that the loading condition in the one case is wholly dynamic and in the other case is combined static and dynamic, and the types of mechanical behaviour under static and dynamic conditions are quite different. Karnes (100), Ripperger (101), and Tapley (102) have also made extensive experimental studies of the mechanical behaviour of metals at high rates-of-strain (see also Refs. 103 and 104). In Karnes' work, short cylindrical specimens of lead and copper were supported at one end by a pressure-bar and were impacted at the other end by a steel projectile. Agreement was found with predictions of the rate-of-strain independent theory only for sufficiently low velocities of impact and away from the impacted end. Experimental values of either stress or strain, but not of both stress and strain, could be made to agree with the predictions of Malvern's quasi-linear rate-of-strain dependent theory for a suitably chosen value of his rate constant $k$. Karnes also concluded that any satisfactory analysis should include the effects of lateral inertia (see also, for example, Ref. 99 for a discussion of the circumstances in which such effects are important). Ripperger's work concerns experimental studies of the dynamic compressive yield-stress of tellurium lead and pure copper; the results indicate that tellurium lead is not essentially rate-of-strain sensitive whereas copper is rate-of-strain sensitive and that there is also an apparent size-effect. Tapley's experimental work with copper bars was similar to Karnes' but particular attention was given to the inclusion in analysis of effects due to lateral inertia. In this way, the discrepancies between experiment and the rate-of-strain dependent theory could be somewhat reduced. It should be noted that the experimental data obtained by Karnes (100) and Tapley (102) were analysed in terms of a particular theory of plastic stress wave propagation (viz. Malvern's), whereas

Kolsky and Douch's (99) experimental data permitted analysis independently of any such an assumption. Hunter and E. D. H. Davies (105) have given a detailed theoretical account of the dynamic compression testing of solids by techniques involving the use of a split pressure-bar. The specimen was in the form of a disk and due attention was given to the complicating effects of lateral inertia and interfacial friction. The experimental data reported included dynamic stress-strain curves, with loading times of the order of 20 microseconds, for annealed pure aluminium and copper. Some dependence of these data upon the specimen thickness was found.

Davis and Hunter (106) have recently given an account of work concerning the assessment of the rate-of-strain sensitivity of metals by means of the correlation of experimental data for conical indentation tests made under static conditions and dynamic conditions with loading times of the order of milliseconds. The empirical approach made was simple and was aimed to provide only useful design data rather than basic physical data.

Few studies with attention to rate-of-strain effects appear to have been made of situations other than those of longitudinal plastic stress wave propagation along bars. However, mention should be made of work by P. S. Symonds and his collaborators on the impact loading of beams. Thus, Bodner and Symonds (47) have found that certain discrepancies between theory and experiment, for cantilever beam specimens made of mild steel and aluminium, could be considerably reduced through an empirical account of the effect of rate-of-strain on the relationship between bending moment and curvature.

Other work concerning very high rates-of-strain in metal plates achieved under explosion conditions is discussed in the next Section.

The limited extent of investigations and the considerable variations in test conditions and the metals chosen for experimentation preclude the derivation of general, firm quantitative conclusions from the above studies. However, there is clear evidence that at least some metals are inappreciably time-dependent in their mechanical behaviour over certain ranges of rates-of-strain. On the other hand, there is clear qualitative evidence that at least some metals are appreciably time-dependent in their mechanical behaviour; in extreme cases, the stress-strain curve is significantly raised throughout its entire length. The increase in yield strength achieved under dynamic conditions is not greatly sensitive to relatively large changes in the rate-of-loading but the ratio of the dynamic and static yield-strengths decreases rapidly with increase in the static yield-strength. Although Malvern's rate-of-strain dependent theory appears to be in broad agreement with certain trends, this theory may not be sufficiently realistic in its simple quasi-linear form and without due account of complicating effects such as lateral inertia and interfacial friction. Furthermore, it must be remembered, as von Kármán and Duwez (32) have remarked, that it is not logical to attempt to provide basic rate-of-strain data from the analysis of results of experiments involving plastic stress wave propagation in the complete absence of a physical theory. The viewpoint that further progress in understanding plastic wave propagation in crystalline solids is dependent upon the formulation of the laws pertaining to the generation and motion of dislocations is discussed by Simmons, Hauser, and Dorn (107) and Campbell, Simmons, and Dorn (108). This work, which includes a unified treatment of the Taylor-von Kármán-Rakhmatulin rate-of-strain independent theory and the Malvern rate-of-strain dependent theory, provides a basic physical approach which clearly indicates reasons for the failure of the former theory and also indicates limitations of the latter. The progress made in understanding time-dependent phenomena in crystalline solids is not given further attention here. However, it may be noted that plastic flow is both temperature and rate-of-strain dependent and involves the operation of processes of thermal activation (see Ref. 64). In many cases, high rates-of-strain and low temperatures produce qualitatively similar effects.

## HIGH-AMPLITUDE STRESS LEVELS

Under surface loadings generated by high-speed impact between a projectile and a target or by an explosion at the surface of a target, anelastic waves are propagated under conditions of extremely high amplitude stress and high rate-of-strain. In typical circumstances, peak stresses of order 300 kilobars may be achieved in times of order $10^{-7}$ sec. and there occur rates-of-strain of order $10^{6}$ sec.$^{-1}$, and then the dependence of the bulk rigidity modulus upon stress and of plastic yielding upon rate-of-strain markedly affects stress-wave propagation occurring in the target. For such situations, realistic constitutive equations are undoubtedly very complicated and considerable differences in the features exhibited can exist between one metal and another. The surface loadings are not known simply from the initial conditions but depend upon the details of the interaction between the target and the load-producing agency; however, in certain cases quite good estimates of these loadings or of their impulses appear to be possible (see Ref. 7). It should be noted that the presence of free boundaries to the target can lead to complications in the stress-wave pattern and to intensification of stress amplitudes due to the focusing of waves (e.g. near corners); and, in particular, material fracture (as with spalling) may occur under reflected tensile stresses. The physical behaviour of metals under conditions of high-amplitude stresses is one of very considerable complexity. The situation envisaged is such that the stress amplitude exceeds by orders of magnitude the numerical value of the initial yield-stress y, being rather more comparable with (or even in excess of) the numerical value of the Young's modulus $E$.[10] The cohesive strength of the material may then become of almost negligible significance in governing the nature of the motion, and the mechanical behaviour of the material can be taken to approximate (although only so long as the stress-amplitude levels are maintained) that of a fluid in which rate-of-strain and thermal-conduction effects are important only where rapid spatial changes in particle velocity occur. The validity or otherwise of this approximation and its implications (as discussed below) is not yet generally established. The effects of high pressure and high temperature on the yield strength act in opposition, such that in a strong shock the resultant effect may be a considerable increase in the compressive yield strength. In this case, there can be significant departures from quasi-fluid mechanical behaviour with resulting complications in stress-wave propagation phenomena.[11] The fact that the bulk and shear rigidity moduli of metals are considerably increased at high pressure has the important implication that higher-amplitude stresses are propagated more rapidly than lower-amplitude ones and hence the leading part of a stress pulse being propagated becomes progressively steeper and, ultimately, almost vertical. Any continuous type of stress wave tends to build up to a vertical front, and then a shock wave develops (see previous discussion in the Introduction). General investigations at high-amplitude stress levels are therefore concerned particularly with the formation and propagation of shock waves, their interaction with continuous waves, and with their breakdown, under conditions of decreasing levels of stress amplitude (caused by dissipation of mechanical energy, due to thermal changes and to structural changes in crystal grains, and in some cases by geometrical divergence), into normal (i.e. low-

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[10] For a high-tensile steel, y = 9.3 kilobars (60 tons/in.$^2$), representatively, and $E$ = 2.1 megabars (13,400 tons/in.$^2$).

[11] For further discussion, see, for example, Ref. 120.

amplitude) plastic and elastic stress-waves when the cohesive strength of the material re-asserts its importance in governing the mechanics of the deformation. It should be noted that the passage of a shock wave through a polycrystalline metal results in physical changes that may affect the subsequent mechanical behaviour (see Ref. 6). In situations involving such extreme complexity, it is not surprising that comparatively little progress in investigations has as yet been achieved.

In proceeding, it is convenient to consider separately studies thay may be classed, rather broadly, according as only either quasi-fluid mechanical behaviour or quasi-fluid and solid mechanical behaviour is involved. The first type of mechanical behaviour is, as will be seen, generally only an approximation to the second type which is the true mechanical behaviour exhibiting much physical complexity, but, nevertheless, investigations made on the former basis (sometimes assuming incompressible flow) have proved to be extremely useful in permitting the limited treatments of certain physical situations.

### Incompressible Quasi-fluid Mechanical Behaviour

The transition in the mechanical behaviour of a metal from that of a normal cohesive solid to that of an apparently cohesionless fluid has been found to occur in a number of physical situations.

For example, if a small metal projectile impacts with a thick target of (say) the same metal at sufficiently high velocity (approaching the order of the bulk elastic stress wave velocity), then a crater of more-or-less hemispherical shape is produced in the target and the original projectile undergoes extremely severe distortion into a thin sheet that lines and coheres to the crater surface. This phenomenon is observed, for example, with a 0.3 gm. spherical aluminium pellet (0.6 cm. diameter) impacting with a thick aluminium plate at 3000 m./sec., the crater diameter being 2 cm. Very many studies of such high-speed impact have been made, and the stress amplitudes that occur during the impact process normally appear to enforce a high degree of quasi-fluid mechanical behaviour on the part of the projectile and, to a lesser degree, of the target (see, for example, Refs. 6 and 109).

An even more striking example of the apparently quasi-fluid mechanical behaviour of metals at high-amplitude stress levels occurs with the so-called shaped charge. This is essentially a device in which a cavity, often cone-shaped, in the surface of a block of high explosive is lined with a thin metal sheet. If the explosive is detonated, the resultant effect of the extremely high pressures of the gaseous explosion products is to cause the metal liner to collapse inwards to form, at least partly, a thin quasi-fluid jet (although sometimes this is more correctly described as fragmented) moving forwards at high velocity. As an illustration, a typical conical liner made of copper is observed to break up separately into a slug and a jet, moving forwards at velocities of the order of 500 to 1000 m./sec. and 7000 m./sec., respectively. The precise structure of the jet varies from one liner material to another: thus, the jet is in the form of a fine spray for the case of lead, a more-or-less continuous jet for copper, and a particulated jet for steel, and these variations may be ascribed mainly to the differences in strength and ductility of the materials. The penetration of the jet produced from a shaped charge into a thick metal target is a remarkable phenomenon: the main feature is the rapid formation of a very deep and narrow hole surrounded by local plastic deformation. To a first approximation, the mechanics of jet formation from a shaped charge and of jet penetration into a target are described in terms of fluid dynamics theories, although certain important aspects of penetration necessarily involve attention to the strength properties of the target itself.

The situations just described have attracted much attention, particularly from the viewpoints of terminal ballistics, impact between space-vehicles and micro-meteorites, and, to a lesser extent, lunar and terrestial crater formation by meteorites.

Birkhoff, MacDougall, Pugh, and Taylor (110) have discussed the mechanics of jet formation produced by a shaped charge from a hydrodynamic (i.e. incompressible perfect fluid) viewpoint. Their theory is most simply developed for a wedge-shaped liner rather than for the more practical case of a cone-shaped one when there is the complication of wall-thickening that ensues as the liner material converges towards the axis of cylindrical symmetry. Essentially, the violent explosion-pressures collapse the cone inwards, this process starting at the apex and proceeding along the axis; the resultant collision (near the axis) of the now more-or-less fluid-like material divides the converging flow into two axial jets, and, relative to the forwards-moving collision-point, the greater part of the material moves backwards to form the slug whereas the remaining smaller part moves forwards to form the jet. Under certain simplifying assumptions—including, particularly, those of hydrodynamic flow and steady motion (relative to the moving collision-point)—an extremely simple theory of jet formation proved possible. Various extensions of this simple theory of Birkhoff et al. have been made, more particularly to take into account departures from steady motion, but nevertheless the limited treatment thereby provided of the mechanics of jet formation has proved remarkably successful (for further discussion, see Refs. 6 and 111). In comparison, the satisfactory treatment of the mechanics of jet penetration into a metal target is much more difficult, this being due mainly to the need to include some account of the strength properties of the target. Pack and Evans (112 and 113) and also others (see Ref. 111) developed a hydrodynamic theory of jet penetration under steady-motion conditions and included some empirical account of target strength and of the secondary penetration associated with the slug. Attention was given primarily to a determination of factors affecting the depth of penetration, discussion of cavitation volume being based on an empirical application of work by Bishop, R. Hill, and Mott (see Ref. 25) concerning static deep-punching in metals. A remarkable prediction of Pack and Evans' simple theory was that the depth of penetration into a given target depended only upon the length and the density of the jet, being quite independent of jet velocity. The true circumstances of jet penetration are of course rather different from those assumed by Pack and Evans, and much work has been done in order to develop improved treatments, particularly in respect of unsteady-motion conditions and of target strength (for further discussion, see Ref. 111).

### Compressible Quasi-fluid Mechanical Behaviour

For most metals, under static pressure conditions, the bulk rigidity modulus increases with the pressure and the finite deformation that occurs is usually recoverable; however, at low pressure, some hysteresis is sometimes found, and at sufficiently high pressure, phase (polymorphic) changes, due to the transition to more stable crystalline forms, are known to occur in certain cases (see, for example, Ref. 114). Significant changes in the bulk rigidity modulus and the density only occur at pressures well in excess of the initial yield-stress (e.g. the density of aluminium is increased by 10 per cent at a pressure of 100 kilobars, approximately). It is also known that the shear rigidity modulus, too, increases with pressure, although the changes are much smaller than those occurring in the bulk rigidity modulus. Bridgman (16) made the first systematic accurate quantitative measurements of the variations in the densities of metals with static pressure up to the order of 30 kilobars (and, in certain cases, 100 kilobars). The stress-amplitude levels that can be produced in thick metal targets by surface explosions are of an order of magnitude higher, say about 300 kilobars. At such stress-amplitude levels, departures from conditions of simple

pressure (not steady or necessarily uniform) due to the cohesive strength of the material are generally taken to be quite unimportant (but see previous remarks) so far as the current mechanical behaviour is concerned. In other words, it is supposed that the material behaves rather like a compressible fluid, although clearly the thermodynamic conditions are not isothermal as is the case in Bridgman's experiments. Improved laboratory techniques now available enable static pressures in excess of those obtained by Bridgman to be achieved (see, for example, Ref. 115). The fact that dynamic pressures of very much higher values up to the order of 4 megabars (see References given below) can be achieved by explosion techniques has been of much interest in relation to studies of the behaviour of metals at very high pressures. Much of the experimental work done has related to the determination of equations of state for metals at very high pressures (see, for example, Ref. 17) and, to a lesser extent, to studies of high-amplitude stress-wave propagation in metals.

Pack, Evans, and James (116) made one of the first detailed investigations of the propagation of shock waves at high-amplitude stress levels in metals. The reason for the occurrence of such shock waves has already been stated. A further fact of importance is that the shock wave velocity increases with the stress-amplitude level involved and, if the level is high enough, then this velocity can exceed the elastic bulk wave velocity. In other words, although polymorphic changes may intervene, it is apparently always possible for shock fronts to be propagated at velocities in excess of the elastic wave velocities applying to small deformations. The question, in any particular case, is simply whether or not the stress-amplitude level is sufficiently high for this circumstance actually to be realized. Pack et al. found experimentally, using explosion techniques, that shock waves travelling at velocities in excess of elastic stress-wave velocities occurred in lead but not in steel, and this situation was due to the different mechanical properties of these materials and to the stress-amplitude level not being high enough in the case of steel. In order to attempt some comparison between the experimental results obtained and theoretical predictions, an equation of state extending to high pressure was needed, in the absence of the requisite pressure-density data from experiment. A semi-empirical formula was derived for the relation between the static pressure $p$ and the density $\rho$ (and hence between the bulk rigidity modulus $K$ and either $p$ or $\rho$) under reversible isothermal conditions. The modulus $K(p)$ was taken to increase with increasing $p$ according to a two-parameter formula derived from quantum-mechanical considerations and fitted empirically to Bridgman's experimental data obtained under reversible isothermal conditions. This formula provided an extrapolation in pressure from about 30 kilobars up to about 300 kilobars, representative of explosion conditions, the possibility of phase changes of course being excluded. In this way, the theoretical predictions of values of shock-wave velocities were found to be in reasonable accord with the experimental results obtained. Very many experimental investigations have been made of the compressibility of metals at high pressures up to at least 300 to 400 kilobars; the pressure-density relations determined experimentally are known as Hugoniot relations, the thermodynamic conditions not being exactly isentropic. Thus there are differences, their magnitude depending upon the pressure level, between equations of state appropriate to reversible isothermal, reversible adiabatic (or isentropic), and Hugoniot conditions. However, given certain thermodynamic data, it is possible to deduce interrelations between these various equations of state, and in this way empirical pressure-density relations have been deduced for the other conditions from the experimentally-obtained Hugoniot data. Rice, McQueen, and Walsh (17) have given a comprehensive summary of work relating to the compressibility of metals (see also Refs. 111, 117, 118, 119, and 120). Very extensive experimental data are now available on the compressibility of metals at high pressures and these data are of course essential for quantitative theoretical studies of high-amplitude stress-wave propagation.

The tensile yield strength of metals at high rates-of-strain, such as occur under explosion conditions, is also a subject of much interest, particularly in connection with fracture investigations. Costello (121), for example, has found that at rates-of-strain of the order of $10^4$ sec.$^{-1}$, achieved through use of explosion techniques, the ratio of the dynamic tensile (reverse) yield-strength to the static tensile yield-strength is 2.9 for mild steel and 1.5 for Vibrac steel. His work showed some indication that the Cottrell-Bilby theory of yielding (see Ref. 64) did not apply at these high rates-of-strain, possibly due to twinning.

A number of other studies directly concerning shock waves in metals has been made. Pack (122) has discussed the nature of the reflected and transmitted waves following explosive detonation at the surface of a block of elastic material. Allen, Morrison, Ray, and Rogers (123) have discussed the steady supersonic flow of copper, treated as a compressible fluid, past an infinite wedge of rigid material and have considered an application to terminal ballistics. Erkman (124) has discussed the formation of oblique plane shock waves in aluminium and copper. Other recent work concerning plane shock waves is that of Duvall (125) who gives an introductory survey of research on the properties and applications of shock waves in inert materials; of Erkman (126 and 127) on the decay of oblique shock waves and spalling produced in aluminium and on the smooth spalling and the polymorphism of iron; and of Fowles (128) on the Hugoniot equation of state for aluminium at pressures from 25 to 50 kilobars. Very little attention appears to have been given to shock waves in metals other than plane ones (see, however, Refs. 15 and 19). In a different field, Gilvarry and J. E. Hill (129 and 130) have treated the formation of lunar and terrestial craters from the viewpoint of the impact by large meteorites, metallic-like materials being assumed, on the basis of fluid mechanical behaviour involving plane shocks. The general interest of this last type of work in connection with the early stages (viz. those involving quasi-fluid mechanical behaviour) of crater formation due to hypervelocity impact is clear.

## Solid and Quasi-fluid Mechanical Behaviour

Due to the very considerable physical and mathematical complexities, comparatively little attention has so far been given to more realistic theoretical treatments (i.e. with account of both solid and quasi-fluid types of mechanical behaviour) of situations involving high-amplitude stresses. However, some detailed investigations have been made of the propagation of high-amplitude stress pulses under one-dimensional plane-strain conditions. Thus, Broberg (131 and 7) has given a somewhat empirical treatment of this situation for a plate of finite thickness with due attention to the reflection of waves, particularly from the viewpoint of the spalling of materials by high explosives. A more comprehensive theoretical discussion of high-amplitude stress-wave propagation in thick steel plates is due to Tupper (132).

Morland (59) has discussed the same situation for a semi-infinite plate made of an aluminium alloy, the surface being subjected to a prescribed smooth loading-unloading pressure pulse broadly representative of a mechanical impact situation. This work constitutes a systematic investigation of the stress-wave propagation in the plate due to this type of applied loading. Essentially, the type of mechanical behaviour assumed is analogous to that conventionally adopted for perfectly-plastic, elastic material but with modifications to allow for the dependence of the bulk rigidity modulus upon

pressure that are based upon Pack, Evans, and James' empirical proposal for the form of the pressure-density relation (the limitations of which proposal have already been discussed). The maximum applied pressure considered in Morland's investigation is such that the elastic bulk stress-wave velocity always exceeds the shock-wave velocity. A rather general analytical discussion is given of certain main features and events in the changing pattern of stress waves. In particular, there is discussion of the interactions between continuous waves, the formation of shock waves, and the interactions between continuous waves and shock waves. The stress-wave pattern is a complicated but an interesting one and the Lagrangian characteristics diagram is shown in Figure 5. A

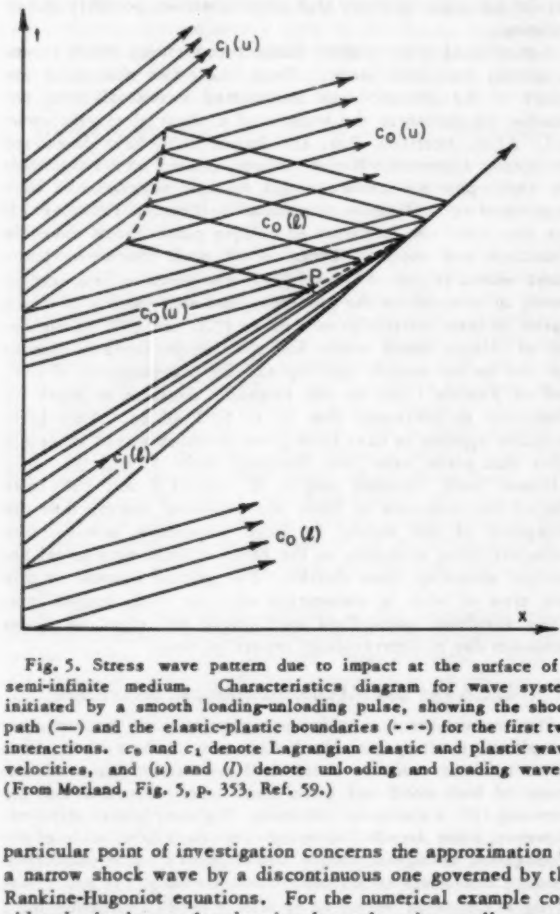


Fig. 5. Stress wave pattern due to impact at the surface of a semi-infinite medium. Characteristics diagram for wave system initiated by a smooth loading-unloading pulse, showing the shock path (—) and the elastic-plastic boundaries (- - -) for the first two interactions. $c_0$ and $c_1$ denote Lagrangian elastic and plastic wave velocities, and ($u$) and ($l$) denote unloading and loading waves. (From Morland, Fig. 5, p. 353, Ref. 59.)

particular point of investigation concerns the approximation of a narrow shock wave by a discontinuous one governed by the Rankine-Hugoniot equations. For the numerical example considered, shocks are found to involve only quite small amounts of thermal energy dissipation but quite appreciable temperature changes can occur. These results are fairly well-known as a characteristic feature of shock waves in metals. Morland's work necessarily involves considerable simplifying assumptions in respect of true mechanical behaviour but otherwise its main limitation is that the application made is to stress-amplitude levels which are low in comparison with those obtaining under, say, explosion conditions. However, the investigation made is particularly notable for its detailed mathematical analysis.

## CONCLUDING REMARKS

In the last two decades, physical phenomena associated with the propagation of stress waves beyond the limit of linear elastic behaviour in metals have been given much and increasing attention from experimental and theoretical viewpoints. The intrinsic difficulties in obtaining accurate physical data and in furthering the development of useful and realistic theoretical studies are only too well known to investigators. In any detailed quantitative study of nonlinear stress-wave propagation in metals, a rather severe compromise is normally necessary between physical realities and mathematical complexities. Certainly, under conditions extending to high rates-of-strain and to high-amplitude stresses no very great quantitative precision is yet to be expected in theoretical studies. Nevertheless, such studies can help to probe and to elucidate the nature of the physical situation while this remains but imperfectly understood. Unfortunately, it is from this viewpoint that many theoretical studies undertaken merit adverse criticism. In many situations of interest, sufficiently realistic or useful theoretical studies must be considered to lag behind experimental ones although the exact interpretation to be given to the experimental data obtained is not always too clear.

Much useful basic experimental research into nonlinear stress-wave propagation in metals can still be done with the use of relatively simple and inexpensive facilities (see Ref. 5). The reason for this is that our ignorance of the extremely-varied mechanical behaviour of metals is so profound that precise measurements of physical data, say to better than one per cent accuracy, are seldom needed. Even if more precise measurements are made, the results obtained will often lack a general significance because the variations in properties over a number of test specimens nominally (or commercially) identical may be quite large. Nevertheless, such experimental research is seldom easy, and this is certainly true of that directed towards the study of rate-of-strain effects. At the present time, our knowledge of constitutive equations with account of these effects is seriously deficient and there is much need of extensive interrelated experimental and theoretical studies. It may be noted here that Lighthill (133) has given an admirable disquisition upon the place of mathematical methods in research into scientific subjects. On the theoretical side, much more explicit recognition needs to be given to the atomic structure of metals. The construction of constitutive equations is likely to involve the use of certain mathematical techniques involved in the rather general investigations now being made of such equations for solid continua (see, for example, Refs. 134 and 135). Constitutive equations must ultimately be in sufficient accord (i.e. for most practical physical or engineering purposes) with experimental data and their precise form will be dependent upon the physical circumstances. Thus, the predictions derived from theories of continuum mechanics based upon general constitutive equations appear likely to be limited, specialization then being required in order to proceed further. In general, the mathematical problems ultimately requiring solution for practical purposes are inherently of a nonlinear nature, and when analytical techniques are either not possible or limited in power and range, recourse must necessarily be made to computational techniques, often based upon characteristics theory[12] and normally with the use of electronic computers.

From the foregoing, it is quite clear that much complementary experimental and theoretical work will be needed in order for more comprehensive mathematical-physical theories of the present subject to be developed. Where there are unexplained discrepancies between experimental data and theoretical predictions, these must of course be ascribed to misinterpretation made in experimental work or to unwarranted simplification of the true physical situation made in theoretical work.

[12]This procedure is straightforward for two-variable problems but is more difficult for three-variable problems (see Ref. 136).

The development of theories that include effects due to rate-of-strain is of paramount importance, and, in fact, such theories may be regarded as central ones in the general theory. This and other developments are unlikely, however, to be particularly easy and, apart from purely phenomenological approaches based upon experimental observations of bulk mechanical behaviour, they involve the challenging task of bridging, or at least narrowing, the gap between work in areas of solid state physics and of continuum mechanics: thus, the atomic nature of anelastic phenomena will ultimately need to be adequately incorporated in theories.


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## Physics, General

**Book—2831. Szabo, I., Repertory and examples in technical mechanics [Repertorium und Ubungsbuch der technischen Mechanik],** Berlin, Springer-Verlag, 1960, vii + 273 pp. DM 24.

The book—as an addition to the well-known texts [see AMR **12**(1959), Revs. 4841, 4842, 4843] of the same author—is written for all engineering students. Included in the introduction to each section of problems is a short review of the principles to be used which should also make the volume valuable for the engineer engaged in mechanical or structural engineering.

The selection of the problems as well as the solutions—given within the text in complete derivation—shows the great teaching experience of the author. The book covers the usual topics, such as: statics of rigid bodies, strength of materials, kinematics and dynamics. The section covering strength of materials gives the fundamental laws of theory of elasticity and their use is demonstrated on a number of very illustrative problems in theory of plates, bending and buckling of circular rings and tubes, etc. Reviewer welcomes that, in the section about vibrations, the continuous bodies, i.e. strings, membranes, beams and plates, are covered by a very representative number of problems. Also within the same section an extensive treatment of liquids and gases, including such topics as Stokes-Navier equations and conformal mapping, is carried out.

The presentation of basic theory and problems by means of a very clearly formulated text—quite often illustrated by sketch of the system—is very well done. The solutions are derived with continuous reference to the basic laws stated at the beginning of each section.

Reviewer believes that the list on the subject literature—given at the end of the text—should be extended in forthcoming editions, then covering the works of Prandtl, Sommerfeld and other well-known authors. The reviewer further questions if the use of two different symbols, i.e. a capital *S* and a regular integral symbol, is justified for indication of integration. The selection of two different symbols for the same operation will hardly be understood by a reader not familiar with the author's earlier books [AMR **12**(1959), Rev. 4842] on the subject.

The unusual pedagogical talent of the author has given the reader more, by far, than just a collection of problems. The volume is strongly recommended to any student or engineer in the field of applied mechanics.

A. Slibar, Germany

**Book—2832. Del Vecchio, A., Dictionary of mechanical engineering,** New York, Philosophical Library, 1961, 346 pp. $6.00.

This book is written for the practicing engineer, the student, and teachers engaged in the fields of architecture, automatic controls, engineering mechanics, fuels and combustion, and power plants. Some definitions are also included in the related fields of electricity, heat treatment of metals, mathematics, and welding.

From author's preface

# Analytical Methods in Applied Mechanics

(See also Revs. 2841, 2844, 2852, 2855, 2857, 2861, 2869, 2887, 2895, 2912, 2913, 2914, 2915, 2919, 2922, 2952, 2963, 2978, 2983, 3046, 3056, 3149, 3175, 3190, 3212, 3257, 3287, 3292, 3319, 3359)

**Book—2833. Dehn, E., Algebraic equations; an introduction to the theories of Lagrange and Galois,** New York, Dover Publications, Inc., 1960, xi + 208 pp. $1.45. (Paperbound)

This new Dover edition is an unabridged and corrected republication of the work first published in 1930. Ed.

**2834. Dvorak, J., The application of a matrix calculation to the determination of the rigidity of heat supply lines** (in Russian), *Inzhener. Fiz. Zh.* **3**, 6, 126–137, June 1960.

Article deals with the calculation of forces and moments acting on the anchorings of piping systems of non-current types. Author quotes the results of his previous work in which he derived, on the basis of the differential geometry of space curves, a symmetrical matrix of influence coefficients relating the vector of generalized deformation to the vector of generalized force.

Application of this method is illustrated on the calculation of a steam piping with restrained ends the center line of which is represented by a cylindrical helix.

The second part of the work presents the solution of ramified piping with non-anchored junction point. The results are valid for any number of branches the center lines of which have the form of analytic space curves converging into one point.

The method is based on the determination of inverse matrices belonging to the individual branches of the system. From the equilibrium conditions at the junction point the components of the displacement caused by the forces acting on the anchorings are determined. After substituting these components into the deformation equations the forces acting at the ends of the branches close to the junction point are calculated.

The treatise represents a generalization of the method indicated in Jürgensonn's book "Elastizität und Festigkeit im Rohrleitungsbau," Berlin, 1953! J. Valenta, Czechoslovakia

**2835. Crossley, F. R. E., and Germen, U., A method of numerical evaluation of a large determinant,** *ASME Trans.* **82**E (*J. Appl. Mech.*), 2, 350–351 (Brief Notes), June 1960.

When solving problems in the vibration of a close-coupled system there arises the need to expand a tridiagonal determinant. This paper contains a very quick and accurate means of accomplishing this expansion, which can readily be adapted to computer programming. From authors' summary

**2836. Garabedian, P. R., Partial differential equations with more than two independent variables in the complex domain,** *J. Math. Mech.* **9**, 2, 241–271, Mar. 1960.

In their canonical form, the elliptic and hyperbolic partial differential equations in two variables are linked by complex domain. Author pursues this idea to higher space.

Upon extending into complex domain: $x_{m+1} \to it$, Green's formula for Laplace equation in $(m+1)$-space goes over to Cauchy's problem for wave equation in $m$-space. The integral surface in this complex domain can then be deformed into a coinciding pair of generalized $m$-disks and a torus about the generalized $(m-1)$-ring, consisting exclusively of branch points of the hyperbolic distance $r$.

For $m$ even, Green's integral over disks add, while that over torus vanishes. The solution is thus an integral over $m$-disk.

For $m$ odd, $r$ appears only in even powers in Green's integral, so disk contributions cancel. However, integral over torus around $(m-1)$-ring now leaves a residue.

That the solution to Cauchy's problem with odd $m$ dimensions is given by an integral over $(m-1)$-space alone, independent of data over $m$-space, is "Huygen's Principle." Difference in the nature of fundamental solutions in odd- and even-numbered dimensional space is explicit.

For elliptic equation, author generalized the image concept to $m$-space, by introducing a "reflected fundamental solution," i.e., the analytic continuation of a given fundamental solution which vanishes on the boundary surface. Applying the contour integration developed above, a general reflection rule was obtained for solutions vanishing over a given analytic surface: The solution at a point is given by a complicated integral over a region on the opposite side, small when the point is near the surface. Thus it was established that an analytic continuation across a reflecting surface depends only on the boundary conditions imposed in the neighborhood, and not on the solution itself.

Reviewer believes the paper a valuable contribution to the study of partial differential equation, and agrees with the author that residual approach has some advantage over Hadamard's finite part principle in its conceptual simplicity. H. S. Tan, USA

**2837. Starzhinskii, V. M., On the method of Lyapunov to estimate the characteristic constant** (in Russian), *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk* no. 4, 46–55, July/Aug. 1959.

This paper is a mathematical analysis of stability of the linear system [1] $x' = a_{11}(t)x_1 + a_{12}(t)x_2$, $x_2' = a_{21}(t)x_1 + a_{22}(t)x_2$, where the coefficients $a_{ij}(t)$ are real, piecewise continuous, and periodic of period $\omega$. It is also assumed that $a_{12}$ and $a_{21}$ are of constant sign and that neither is identically zero. It is also assumed that $\alpha \geqq \beta$ where $\alpha = \int_0^\omega a_{11}(t)\,dt$, $\beta = \int_0^\omega a_{22}(t)\,dt$. By the substitution $x_1 = x \exp \int_0^t a_{11}(s)\,ds$, $x_2 = y \exp \int_0^t \left[a_{22}(s) - \frac{\alpha - \beta}{\omega}\right] ds$, the system is reduced to [2[ $x' = r(t)y$, $y' = q(t)x - ay$, where $q(t)$ and $r(t)$ have constant sign and $a = \frac{\alpha - \beta}{\omega} \geqq 0$. If $V(t)$ is that fundamental matrix of the system [2] which is the identity at $t = 0$, stability is determined by the quantity $A$, which is the trace of $V(\omega)$, divided by two.

The author gives conditions for stability of the solutions of [2] and [1] in terms of the values of $A$. As an example, if $\alpha + \beta < 0$, the solutions of [1] are stable or unstable according as $|A|$ is less than or greater than $(e^{-\alpha} + e^{\beta})/2$.

The principal results of the paper are techniques for estimating $A$. The author makes the additional assumption that $a_{12}(t)$ and $a_{21}(t)$ have opposite sign. The author gives a series due to Lyapunov for $A$, $A = A_0 - A_1 + A_2 - \ldots$ where the terms arise from the solution of the system [2] by successive approximations. A typical sufficient condition for stability of the solutions of [1] is $0 \leqq \alpha < -\beta$,

$$A_2 + \tfrac{1}{2}(1 - e^{-\alpha})(1 - e^{\beta}) \leqq A_1 \leqq \tfrac{1}{2}(1 + e^{-\alpha})(1 + e^{\beta}).$$

W. S. Loud, USA

**2838. Kulikov, N. K., Conditions of existence and determination of parameters of periodic motions of autonomous systems** (in Russian), *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk* no. 4, 56–62, July/Aug. 1959.

This is a study of periodic solutions of $n$th order ordinary differential equations which are autonomous, i.e. of the form $\varphi(x^{(n)}, x^{(n-1)}, \ldots, x', x) = 0$. The author extends a technique developed in an earlier Russian paper [*Izv. Vyssih. Učebnyh. Zavedenii, Ser. Matematika*, **4**, no. 5, Kazan', 1958] in which he was able to find either exact solutions or approximate solutions. His technique is essentially to approximate the differential equation by an equation with constant coefficients, and to require that appropriate relations

be satisfied. Illustrations are given with a linear equation with constant coefficients and some simple nonlinear equations. The examples that are nonlinear are of second order, and the method of the author seems to be equivalent to the method of van der Pol, which is well known in analytical mechanics.

W. S. Loud, USA

## Computing Methods and Computers

(See also Revs. 2835, 2858, 2863, 2869, 2901, 2927, 2950, 3048, 3111, 3229, 3231)

**2839. Lowan, A. N., On the propagation of round-off errors in the numerical treatment of the wave equation,** *Math. Comput.* **14**, 71, 223–228, July 1960.

An upper bound is given for the round-off errors in the numerical solution by finite differences of the wave equation

$$u_{tt} = c^2 u_{xx} \text{ in } 0 \leqslant x \leqslant a,\ t > 0,$$

with the initial and boundary conditions $u(x, 0) = f(x)$, $u_t(x, 0) = g(x)$, and $u(0, t) = u(a, t) = 0$. The upper bound after $n$ time steps of the calculation is

$$E_n^* = [(n+1)(n+2)/2]\, M^{1/2} \delta^*$$

where $M$ is the number of interior mesh points. If the solution is carried out by using the explicit finite difference analog of the wave equation, then $\delta^* = (3/2)\, 10^{-p}$, when the computations are carried to $p$ decimal places. If the implicit finite difference analog is used, $\delta^*$ depends on the method used for solving the system of implicit equations.

H. G. Landau, USA

**2840. Miyakoda, K., Numerical calculations of Laplacian and Jacobian using 9 and 25 gridpoint systems,** *J. Meteorol. Soc. Japan (II)* **38**, 2, 94–106, Apr. 1960.

Finite-difference formulas for use in numerical weather prediction need special care to avoid instability in the time step-by-step process, and truncation errors of certain kinds are more important than others. A nine-point formula for the Laplacian can be obtained by combining linearly two five-point formulas, one of which corresponds to a rotation of axes. The twenty-five point formula involves other rotations, and so on. The coefficients of the linear combination are chosen to make certain terms in the truncation error vanish or to have suitable behavior.

Certain functions representative of typhoons and cyclones are analyzed using various formulas, to show the effect of "orientated" truncation error. Further examination to find the standard deviation, on a large grid, between analytical and approximate values of the Laplacian indicates an optimum form for a nine-point operator, which turns out to be somewhat different from that associated with minimum orientational error.

Similar numerical experiments with approximations for the Jacobian, which is involved in the weather prediction equations, suggests that all nine-point formulas have large errors, and these are considerably reduced with a suitable 25-point formula.

L. Fox, England

**2841. Miyakoda, K., Test of convergence speed of iterative methods for solving 2- and 3-dimensional elliptic-type differential equations,** *J. Meteorol. Soc. Japan (II)* **38**, 2, 107–124, Apr. 1960.

First section discusses tolerable size of residual which suitably limits error of result, and numerical tests indicate theory has satisfactory safety factor. Theoretical investigations of rate of convergence and optimum values of parameters in over-relaxation devices with five-point formulas repeat known results for Poisson's equations and extend theory to Helmholtz equation. Numerical tests confirm predictions in two dimensions and, less satisfactorily, in three. Some known properties of nine-point formulas are also verified by numerical experiment. Normal derivative boundary conditions are treated, using a formula with rather large truncation error, and numerical tests give a somewhat larger optimum over-relaxation factor than is predicted by theory. Methods of Richardson and of Peaceman and Rachford are also discussed and tested, and conclusions are that alternating direction method is fastest but procedurally difficult with complicated boundary conditions, for which case accelerated Liebmann is preferred. The various procedures tested are given in FORTRAN language in Appendix. Paper has little novel theory but interesting numerical results.

L. Fox, England

**2842. Nitsche, J., and Nitsche, J. C. C., Error estimates for the numerical solution of elliptic differential equations** (in English), *Arch. Rational Mech. Anal.* 5, 4, 293–306, 1960.

The equation considered has the form $au_{xx} + 2\, bu_{xy} + cu_{yy} = f$, $a$, $b$, and $c$ being continuous functions in the square $0 \leqslant x, y \leqslant 1$, for which the ellipticity of the equation is uniform, $f$ a square-integrable function over the same region; the boundary condition is $u = 0$. The approximate solution, obtained by difference method, is defined on the nodes of the square mesh. The error is estimated as dependent on the mesh width $h$ and moduli of continuity for the coefficients $a$, $b$, $c$ as well as for $f$ (or its appropriate means, if $f$ is discontinuous). Several lemmas are needed in derivation of the final estimates. In the simplest case, where $a$, $b$, $c$, and $f$ fulfill Lipschitz conditions, the error is $0(h^{2/3})$.

P. Laasonen, Finland

**2843. Nitsche, J. C. C., and Nitsche, J., Error estimates for the numerical calculation of integrals containing solutions of elliptic differential equations** (in German), *Arch. Rational Mech. Anal.* 5, 4, 307–314, 1960.

Paper deals with the same boundary-value problem and approximation for its solution discussed in preceding review. Instead of the solution function $u$, some integrals of the form $\iint pudxdy$ and $\iint pD^2udxdy$ are studied, where $p$ is a square-integrable function and $D^2u$ is any second-order derivative of $u$. The integrals are approximated by corresponding sums, where $p$ is represented by an appropriate mean and $u$ by the corresponding grid function. Some estimates for the errors are derived.

P. Laasonen, Finland

**2844. Walton, W. C., Jr., Applications of a general finite-difference method for calculating bending deformations of solid plates,** NASA TN D-536, 47 pp., Nov. 1960.

Author reports findings on a new finite-difference method developed by Houbolt [Diss. Zurich 1958; Mitt. no. 5, Inst. Flugzeugstatik u. Leichtbau] and directly applicable to static or simple harmonic flexures of solid plates and potentially useful in other problems of structural analysis. Method is based on linear theory and incorporates principle of minimum potential energy. Full realization of its advantages requires high-speed computing. The natural modes and frequencies of uniform-thickness cantilever plates and uniform free-free beams are calculated. Computed plate frequencies and nodal patterns for the first five or six modes are compared with experiment and other approximate theory; beam computations are compared with exact theory. Method is concluded to be accurate and general in predicting plate flexures. Computing procedure is appended.

From author's summary by M. L. Meyer, England

**Book—2845. Haley, A. C. D., and Scott, W. E., edited by, Analogue and digital computers,** New York, Philosophical Library Inc., 1960, viii + 308 pp. $15.

This book of ten chapters is the joint effort of seven authors. I dislike such joint enterprises as the different authors naturally tend to write on different levels and of course in different styles, and there is a natural tendency toward repetition, as in the present volume, in which magnetic tape units are given two treatments by different authors on pp. 256–259, and pp. 277–280.

However, despite the above canard, I must admit that the authors have packed into 303 pages much more information than is contained in many more pretentious volumes.

The four chapters on analog computers cover only the electronic differential analyzer type of machine, neglecting network analyzers and various other direct analogs. Chapters 3 and 4, both by Dr. M. J. Somerville of Manchester University, are the best, and go into some detail on circuit and system design, but are unfortunately exclusively concerned with vacuum tube circuits.

The six chapters on digital computers vary widely in the level of presentation. By far the best are Chap. 7 "Circuit elements and computer units" and Chap. 8, "Storage" by Dr. R. L. Grimsdale of Manchester. These are well written and vividly cover a great deal of material.

In such a compressed treatment, it hardly seems worthwhile at this stage of the technology to devote five pages to cathode-ray tube memories. This space could have been better devoted, for example, to newer techniques in magnetic core memories, such as the word-selection scheme. Fortunately he presents the Manchester "phase modulation" method recording on magnetic drums, and the use of magnetostrictive delay lines as memory devices, neither of which topics are widely known in this country. Unfortunately nothing is said about the interesting new "Atlas" program for the development of super-speed computers at Manchester.

All things considered, this is a useful and informative book.

C. V. L. Smith, USA

**2846. Isobe, T., and Nihei, T., Automatic plotting of equipotential lines,** Aero. Res. Inst., Univ. of Tokyo, **24,** 9, 211–221, Nov. 1958.

The manual operation of plotting equipotential lines between model electrodes in an electrolytic tank has been made automatic by applying a servomechanism. In the arrangement, the probe, which itself consists of three needles rotatable about the center, is moved in the *X* and *Y* directions by two servo-motors arranged correspondingly. The outer two needles of the probe incessantly seek out the tangent of line and follow up the direction automatically. And in this direction the probe is made to move at a preset constant speed. On the other hand, any deviation of the probe from the line is detected by the center needle, and is corrected continually in the normal direction. By these means, any closed or open equipotential line may be automatically traced out. The detailed constructions, circuits and the performance of the arrangement with a few examples of plot are described.

From authors' summary

## Analogies

(See Revs. 2858, 3012, 3042, 3115, 3252, 3303)

## Kinematics, Rigid Dynamics and Oscillations

(See also Revs. 2831, 2861, 2873, 2874, 2875, 2976, 2978, 3067, 3069, 3287, 3289, 3295)

**2847. Bastin, J. A., An extension of the Newtonian law of gravitation,** *Proc. Camb. Phil. Soc.* **56,** 4, 401–409, Oct. 1960.

An attempt is made to relate such diverse phenomena as kinetic energy, rest mass energy, the size of the universe, the velocity of propagation both of light and gravitation, and the recession of the galaxies. The correlation is made by considering an extension of the Newtonian gravitational law which covers a particular simple case when the two attracting bodies are in relative motion. The extension is treated as postulatory, although in the last section, using the idea of gravitational flux, the assumed gravitational law is shown to be the simplest of a number of possible extensions to the Newtonian law. The paper implies a new approach to special relativity, and author hopes to treat aspects of the work in greater detail subsequently.

From author's summary by J. M. Frankland, USA

**2848. Figueras, H., Discontinuity of invariant differentials of the trajectories of charged particles** (in French), *C. R. Acad. Sci. Paris* **250,** 22, 3567–3569, May 1960.

**2849. Bottema, O., Nonholonomic motion of a plane slab** (in German), *ZAMM* **40,** 5/6, 275–276, May/June 1960.

It is shown that the general nonholonomic condition which can be set up for the plane motion of a rigid disk becomes identical with the nonholonomic condition given by Caratheodory for a sledge, if invariance against the special choice of the coordinates is demanded.

K. Magnus, Germany

**2850. Boillet, P., Extension of the principle of Huygens to a discontinuous medium: the medium considered as an aggregate of atoms** (in French), *C. R. Acad. Sci. Paris* **250,** 20, 3274–3276, May 1960.

**2851. Symposium on the dynamics of the internal-combustion engine,** Instn. Mech. Engrs., Auto. Div. no. 4, 49–110, 1958/59.

**2852. Lemoine, L., Low frequency vibrating table for the dynamic calibration of accelerometers** (in French), ONERA Pub. 69, 27–37, Mar./Apr. 1959.

Author makes an analytical review of accelerations and accelerometer reactions thereto. The complexities of low frequency vibrational accelerations, including wave form, direction, and harmonic effects, and a device which provides low frequency accelerations applicable to effective dynamic calibrations as covered by the title of the article are, in the reviewer's opinion, clearly described and well analyzed.

L. B. Hedge, USA

**2853. Lees, S., Felsenthal, H. D., and Goldberg, E. M., Estimating the roots of the characteristic determinant for multicoupled systems,** *ASME Trans.* **82** D (*J. Basic Engng.*), 1, 87–95, Mar. 1960.

The essential content is well-epitomized in the authors' abstract: "A procedure is developed, based on Cauchy's residue theorem, for bounding the undamped frequency, damping ratio, and real part of the roots of the characteristic determinant associated with a multicoupled system with several inputs and outputs. The method can lead to a locus of the roots as one or more parameters are varied. The underlying theory is developed and a numerical illustrative example is included."

Though this article is of value to all concerned with the dynamics of engineering systems, it may be of particular interest to automatic control-theory specialists. Relative to the latter, and in usual control-theory terminology, the authors have conjoined vibration theory and Nyquist and root-locus plotting to obtain root-trajectories whereon the variable parameter is one of the coefficients of the characteristic determinant pertinent to a linear lumped-parameter multiple-input multiple-out system characterized by a set of constant-coefficient linear integrodifferential equations.

T. J. Higgins, USA

**Book—2854. Chisnell, R. F., Vibrating systems,** London, Routledge & Kegan Paul, 1960, vi + 89 pp. 5s. (Paperbound)

Book discusses two-degree of freedom oscillations in mechanical and electrical systems. It contains a clear treatment of normal modes, Lagrange's equations and forced vibrations. Discussion throughout is at early undergraduate engineering level. Text includes illustrative examples. Problems of conventional type with answers are provided. Book is ideal as supplementary text or for self-study by student. T. P. Mitchell, USA

**2855. Rosenberg, R. M., Normal modes of nonlinear dual-mode systems,** *ASME Trans.* **82** E (*J. Appl. Mech.*), 2, 263–268, June 1960.

This very interesting paper deals with systems of type

$$\ddot{x}_i = -\Sigma a_n x_i^n - \Sigma b_n (x_i - x_j)^n \ (i, j = 1, 2),$$

the polynomials on the right-hand side being odd. The "normal modes" are functions $x_2 = f(x_1)$ which are identically satisfied by periodic solutions $x_i(t) = x_i(t + T)$ of the system. They can be defined as the geodesics of a certain surface characterized by means of the potential function of the system. The knowledge of the normal modes reduces the original two-degree-of-freedom problem to two single one-degree-of freedom problems. Author particularly discusses two special cases for which the function $f(x_1)$ is linear. W. Hahn, India

**2856. Klotter, K., and Kreyszig, E., On a special class of self-sustained oscillations,** *ASME Trans.* **82** E (*J. Appl. Mech.*), 3, 568–574, Sept. 1960.

This paper deals with the "modified van der Pol differential equations" $\ddot{q} - (\operatorname{sgn} q)(\delta/2)q^2 b(q) + \kappa^2 f(q) = 0$, where $b(q)$ and $f(q)$ are suitable functions. It is shown that there exists a unique limit cycle. For the limit amplitude both lower and upper bounds are established in the case of unrestricted values $\delta$. For small values $\delta$ the limit amplitude can be calculated immediately. From authors' summary

**2857. Bradistilov, G., and Boyadjiev, G., Existence of periodic motion of an *n*-fold pendulum when certain roots of its characteristic equation are multiple** (in German), *ZAMM* **39**, 7/8, 284–290, July/Aug. 1959.

In a previous paper by one of the authors (G. Bradistilov) it has been proved that for a system of $n$ coplanar pendulums coupled in series there exist $n$ families of periodic motion provided that none of the roots of the characteristic equation is an integer multiple of any of the other roots. In the present paper the same problem is considered under the assumption that $p - 1$ roots of the characteristic equation are multiples of another root. It is proved that for this root the number of families of periodic motion is equal to the number of real solutions of a system of $p - 1$ algebraic equations of 4th degree containing $p - 1$ unknowns, a system that represents the conditions for periodicity. From authors' summary by K. N. Tong, USA

# Instrumentation and Automatic Control

(See also Rev. 2853, 3299)

**2858. Emschermann, H. H., and Rohrbach, C., Direct electrical integration of closed measuring curves and its application for measuring the damping of metals and for indicating piston engines** (in German), *ZVDI* **103**, 5, 169–176, 1961.

Previous methods for measuring the integral of closed curves (e.g., the hysteresis curve of an alternately stressed elastic-plastic specimen, or the indicator diagram of a piston machine) are usually complicated and time consuming, and are adapted for moderate frequency limit. A novel method is described which is claimed to be applicable to a practically unlimited frequency range, and with which the value measured can be determined (indicated or recorded) during the measurement. Thereby it is possible to obtain the integrated value under varied influencing parameters (temperature, stress, load). The principle consists in converting the "non-time" variable (e.g., stroke, or displacement) into a "velocity-times-time" variable, and using a velocity-responsive electric pickup which converts the velocity into an electric magnitude, the time integral of which can be obtained easily. The theory is explained, and electric circuits are shown, incorporating strain-gage elements with variable resistance. As illustrations, the method is applied to a torsionally stressed test specimen, and to a compressor operated at various output conditions, for both of which oscillograms are shown. K. J. DeJuhasz, USA

**2859. Merkulova, E. P., The problem of optimizing systems which contain essentially nonlinear elements,** *Automation and Remote Control* **20**, 10, 1303–1313, June 1960. (Translation of *Avtomatika i Telemekanika, USSR* **20**, 10, 1335–1344, Oct. 1959 by Instrument Society of America, Pittsburgh 22, Pa.)

Paper deals with control systems with linear and nonlinear constant-parameter elements in forward circuit and feedback path, the nonlinear elements being of relay type or linear over limited intervals. Input signal is sum of uncorrelated signal and noise, a stationary random function of time with rational spectral density. Correcting element is optimized in the sense that the variance of the output is to be a minimum. The procedure given replaces each nonlinear element by a linear one such that the expectation and variance of its output are the same as for the actual nonlinear element. The optimizing condition on the weighting function of the linearized correcting loop results in a problem of the calculus of variations. The weighting function is characterized as the solution of an integral equation which must be solved by successive approximations. Also considered is the case that the input signal contains a time-varying part. Then the procedure becomes even more complicated, and may fail. The required calculations are indicated for two illustrative examples. M. Slud, USA

**2860. Rozenman, E. A., Optimal control of an object with two controlling stimuli,** *Automation and Remote Control* **20**, 10, 1314–1318, June 1960. (Translation of *Avtomatika i Telemekanika, USSR* **20**, 10, 1345–1349, Oct. 1959 by Instrument Society of America, Pittsburgh 22, Pa.)

Author deals with the problem of the optimal transient response of a regulated system at whose input the product of two independent controlling functions acts. This kind of problem is given in particular in the case of electric motors used in automatic devices, the excitation and the armature currents varying simultaneously and independently.

Two questions are examined: to determine the excitation and the armature currents as functions of time so that (1) for a fixed duration of the process the energy dissipation will be minimal, and (2) for a bounded dissipation of energy the duration of the transient response will be a minimum. It is shown that for both cases the extrema are combinations of $\delta$-functions.

Special consideration is given to the fact that infinite controlling stimuli cannot be realized and therefore limiting conditions have to be imposed in addition to the criteria of optimization mentioned before. P. J. Profos, Switzerland

**2861. Hajek, J., Equivalence of three stability criteria** (in German), *ZAMM* **39**, 7/8, 332–333, July/Aug. 1959.

Paper presents a simple proof, without the use of theory of functions, of the equivalence of the three criteria of stability for

polynomials with real coefficients. Starting with criterion by Hurwitz, author shows through manipulating the Hurwitz determinants that the necessary and sufficient condition for the stability of a polynomial of $n$-th degree is that a related polynomial of $(n-1)$-th degree be stable. Since the same conclusion can also be readily arrived at from the criteria of Schur and of Routh, the three criteria are shown to be equivalent. Paper contains a few minor misprints. K. N. Tong, USA

**2862. Vol'pert, E. G., On setting up an operating program for several control systems,** *Automation and Remote Control* **20**, 12, 1626–1629, June 1960. (Translation of *Avtomatika i Telemekanika, USSR* **20**, 12, 1675–1678, Dec. 1959 by Instrument Society of America, Pittsburgh 22, Pa.)

In the control of systems having several inputs, or in the application of one controller to several simple systems, it is natural to use a sampling process.

This brief paper outlines a method for programming the switching necessary using probabilistic methods so that the variables most likely to vary rapidly, or to which the controlled process is most sensitive, are sampled more frequently than the others. It is evident that even in simple systems the frequency of switching can be reduced from that used in sampling at regular intervals with corresponding ease of operation and increased life.

P. G. Kirmser, USA

**2863. Kalyaev, A. V., Computing the transient response in linear systems by the method of reducing the order of the differential equation,** *Automation and Remote Control* **20**, 9, 1141–1150, May 1960. (Translation of *Avtomatika i Telemekanika, USSR* **20**, 9, 1171–1179, Sept. 1959 by Instrument Society of America, Pittsburgh 22, Pa.)

Linear differential equations with constant coefficients as obtained in automatic control systems are reduced by transformations to lower-order equations. This method results in rapid evaluation of response time, overshoot and number of oscillations with good accuracy. Numerical examples with orders reduced by a factor of 2 or 3 are presented. C. R. Freberg, USA

**2864. Leonov, Yu. P., On approximate synthesis of optimum linear detecting systems,** *Automation and Remote Control* **20**, 8, 1039–1048, Apr. 1960. (Translation of *Avtomatika i Telemekanika, USSR* **20**, 8, 1071–1080, Aug. 1959 by Instrument Society of America, Pittsburgh 22, Pa.)

The paper is concerned with determining an optimum filter such that the expression

$$I_1(k) = M\left\{\int_T^t k(t,\tau)\,y(\tau)\,d\tau - g(t)\right\}^2 \qquad [1]$$

has a minimum value. In equation [1] $y(t)$ is the input signal, which is related to $g(t)$ according to $y(t) = g(t) + n(t)$ where $n(t)$ is a random function with a known correlation function and zero mean. A weight function $k(t, \tau)$ is to be determined such that $I_1(k)$ has the desired minimum. It is left to the reader to deduce the definition of $M[f(t)]^2$ used in equation [1]. The minimization is based upon the method of Lagrangian multipliers along with an approximation scheme such as the Ritz method. By means of elementary theory of Hilbert space the author shows that while the exact solution lies in a Hilbert space $H_A$ the approximation method is restricted to an incomplete subspace $H$ such that $H \subset H_A$. It follows that a solution may not exist, so that, as the author states, the existence of a solution must be investigated in each individual case. Comparison is made only with the method of Pugachev, and the subject method is said to converge more rapidly. W. C. Orthwein, USA

**2865. Emel'yanov, S. V., Control of a first order delayed system by means of an astatic regulator and nonlinear correction,** *Automation and Remote Control* **20**, 8, 983–991, Apr. 1960. (Translation of *Avtomatika i Telemekhanika, USSR* **20**, 8, 1009–1019, Aug. 1959 by Instrument Society of America, Pittsburgh 22, Pa.)

This paper is devoted to the commonly occuring case of a system to be controlled where this system is described by a first-order lag in series with a pure delay. The computer part of the controller feeds into an integrating servomotor whose output is the input to the system to be controlled. The optimum response of the system is obtained for step disturbances coming into the system at any one of three points, namely at the system output, at the system input right after the servomotor, and between the pure delay and first-order lag elements. Control functions are constructed to yield near optimum response where the controller depends only on the error and the rate of change of this error with respect to time. The input to the servomotor is taken to be proportional to the error where the value of the coefficient of proportionality is a function of the products and quotients of the error and its derivative. The transfer functions of many physical systems can be approximated by the system transfer function of this paper.

In the opinion of the reviewer this work is a major contribution to automatic control theory. R. Oldenburger, USA

**2866. Ostrovskii, G. M., Increasing the speed of certain automatic control systems by means of nonlinear and computing devices,** *Automation and Remote Control* **19**, 3, 199–209, Feb. 1959. (Translation of *Avtomatika i Telemekhanika, USSR* **19**, 3, 208–217, Mar. 1958 by Instrument Society of America, Pittsburgh, Pa.)

Author extends the method he originally applied to a second-order system [AMR **12**(1959), Rev. 39] to $n$th order systems in which the error $x$ satisfies the differential equation $x^{(n)} + a_1 x^{(n-1)} + \ldots + a_{n-1}\dot{x} + a_n x = 0$. The requirements imposed on the corrective device are (1) damping is small for large errors and large for small errors (to minimize transient response time), and (2) the transient response is not over-controlled. Paper considers a corrective device which sets the coefficients $a_i$ equal to $c_i$ or $b_i$ respectively, $c_i < b_i$, according as error is large or small. The problem is to determine the precise moment for changing the coefficients. Author finds this criterion, obtains a necessary and sufficient condition for monotonic integral curves, and shows how an analog computing device could determine the time to switch. A third-order control system is used as illustration.

M. Slud, USA

**2867. Rozenvasser, E. N., On the stability of nonlinear control systems described by fifth and sixth order differential equations,** *Automation and Remote Control* **19**, 2, 91–102, Jan. 1959. (Translation of *Avtomatika i Telemekhanika, USSR* **19**, 2, 101–114, Feb. 1958 by Instrument Society of America, Pittsburgh, Pa.)

In his now classical monograph, Luré ["Some non-linear problems in the theory of automatic control," Gos. Isdat. Tekh. Teor. Lit., 1951, USSR] gives sufficient conditions (too detailed to reproduce here) for the stability of a nonlinear control system with a single controlling element. He carries out the computations for systems described by differential equations of order up to the fourth. The present paper carries out the similar but more difficult calculations for systems described by differential equations of orders 5 and 6. M. Slud, USA

**2868. Itskovich, E. L., On designing control circuits for objects with pure lags,** *Automation and Remote Control* **20**, 8, 1016–1023, Apr. 1960. (Translation of *Avtomatika i Telemekhanika, USSR* **20**, 8, 1047–1055, Aug. 1959 by Instrument Society of America, Pittsburgh 22, Pa.)

Author develops, analytically, a method for designing a control system for an object with lags acted upon by random disturbances.

The decay time of the correlation function of the quantity to be controlled ($t_0$) is compared with the duration of the transport lag ($\tau$) in the object. In order to construct such a control system, $t_0$ must be greater than $\tau$. The correlation function is used to develop the criteria for a control system to maintain a given average quality in the controlled object. Author shows that this will require the control of certain other disturbing factors if the condition of $t_0 > \tau$ is not met in the basic system. This will result in a multiloop control system.

As an example of use of this method, a control system was designed for controlling one parameter of the output from a rotating cement-roasting kiln. Results are given indicating successful operation was obtained even though the condition of $t_0 > \tau$ was not met in the basic system. R. K. Beach, USA

**2869. Batkov, A. M., Generalization of the shaping filter method to include nonstationary random processes,** *Automation and Remote Control* **20**, 8, 1049–1062, Apr. 1960. (Translation of *Avtomatika i Telemekhanika, USSR* **20**, 8, 1081–1094, Aug. 1959 by Instrument Society of America, Pittsburgh 22, Pa.)

Author considers the basic properties of the impulse response of linear systems with variable coefficients of general form. These properties are then used to compute the correlation function of the output from such systems, when acted upon by "white noise." Using this material as a foundation, author considers the problem of determining the system parameters of a shaping filter such that a prescribed output correlation function is obtained when the system is excited by "white noise." Author then discusses the application of the method of shaping filters to the determination of the dynamic accuracy of automatic control systems operating with random stimuli.

This paper is a valuable generalization of the shaping filter method to include nonstationary random processes.

T. K. Caughey, USA

**2870. Bahniuk, E., and Lee, S.-Y., The design and analysis of a servo valve with flow feedback,** *ASME Trans.* **82** D (*J. Basic Engng.*), 1, 73–80, Mar. 1960.

The valve described consists of a flapper-nozzle stage followed by a spool stage, with feedback to the flapper from a flowmeter, the spool having no spring restraint. The flowmeter is of the "area" type, consisting of a poppet valve backed by a low-rate spring, thus giving a displacement proportional to flow (i.e. load flow from the spool stage). One flowmeter is used for each side of the spool. Flowmeter movement is converted to force by a spring, which feeds back to flapper. It is shown that friction in flowmeter does not produce a dead zone.

Flow feedback linearizes the valve, output flow being independent of load. Main advantage however is that spool accuracy can be quite low, spool overlap can be tolerated (leading to power economy and also allowing larger diametral clearances), erosion effects are unimportant, and gain is independent of supply pressure. Reduction in accuracy requirements offsets extra cost of flowmeter. Reviewer believes these claims to be justified.

Static and dynamic characteristics are analyzed under zero load conditions, and measured performance curves are given. It is shown that stability is dependent on damping of flapper.

The paper is lucidly written and, in reviewer's opinion, represents an important contribution to the hydraulic servo control art.

R. Hadekel, England

**2871. Bodner, V. A., Seleznov, V. P., and Ovcharov, V. E., Contribution to the theory of inertial damped systems with arbitrary period, invariant with respect to maneuvering of the object,** *ARS J.* **30**, 1, 93–97 (Russian Supplement), Jan. 1960. (Translation of *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk, Energetika i Avtomatika* no. 3, 11–18, 1959.)

Paper deals with inertial damped systems immune to maneuvering accelerations. The first section of the paper may be regarded as introduction; it is shown that it is impossible to obtain a damping of such a system by means of internal feedback without destroying its immunity to accelerations (the Schuler theorem).

It is proposed to use the sampled data information from the actual ground position of the object as well as its velocity, acceleration, etc. The information is introduced at the input of the first and the second integrators and also at the input of the gyro platform. Introducing the error signals (of angle and angular velocity) defined as differences between the inertial data and those of the external information, it is shown that the period of the inertial system can be changed relatively to the Schuler period without violating the above theorem. The external information must be in all respects of the sampled data type (its action must take place at the discrete instants and be of short duration). It is preferable to reduce the period (with respect to the Schuler period) in order to eliminate disturbances in a shortest possible time.

In the third section authors investigate the quantitative part of the problem in two cases: (1) errors are independent of time; (2) errors are random functions of time. The first case is simple if one knows the component errors (of the inertial vertical and the information data). In the second case it is necessary to determine spectral densities of component errors; this leads to the determination of the mean square values of errors; the latter are represented as functions of a parameter $k$, the gain in integrating elements. The last section deals with the question of high velocities. It is shown that for a critical velocity the period increases indefinitely; in this case only the initial correction is of importance and the subsequent behavior is of the Schuler type.

N. Minorsky, France

**2872. Ishlinskii, A. Iu., On the autonomous determination of position of a moving object by means of a space gyrocompass, a directional gyroscope and an integrating device,** *Appl. Math. Mech.* (*Prikl. Mat. Mekh.*) **23**, 1, 75–82, 1959. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

The problem of determination of position of an object moving on a spherical earth requires the use of a directional gyroscope and of a device which integrates a system of three nonlinear differential equations of the first order. As these equations do not involve the time variable explicitly, they are autonomous. Author derives these equations of motion, analyzes them and discusses the case when the initial conditions are not exactly satisfied.

The reviewer (who also translated this paper) noticed an inaccuracy. The sentence which follows formula (8) should be replaced by: "Here $V_y$ is the $y$ component of the velocity vector $V$ (the $y$ axis coincides with the vector of angular momentum $H$)."

T. Leser, USA

**2873. Ishlinskii, A. Iu., On the equations of the precessional theory of a gyroscope in the form of equations of motion of the pole in the phase plane,** *Appl. Math. Mech.* (*Prikl. Mat. Mekh.*) **23**, 5, 1153–1163, 1959. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

In the theory of gyroscopes the phase plane is the plane a unit distant from the $xy$ plane and parallel to it, where the origin coincides with the center of the gimbal suspension and the $z$ axis is either vertical, or, as in the case of a gyrocompass, it is directed North. The "pole" (the "representative point") is the point of intersection of the angular momentum vector with the phase plane. The author gives a rigorous justification for the equations of motion of the pole and shows that they are valid only when the forces acting on a gyroscope satisfy a number of conditions.

T. Leser, USA

**2874. Koshliakov, V. N., The theory of a gyrocompass,** *Appl. Math. Mech.* (*Prikl. Mat. Mekh.*) **23**, 5, 1164–1173, 1959. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

Author analyzes the equations of a perturbed motion of a gyrocompass with two gyroscopes coupled by a spring. It is assumed that the gyrocompass is being used at a high latitude (70°–80°), which prevents neglecting certain terms in the equations of motion. The author writes down the differential equations of the perturbed motion in the general case and then solves them for the following special cases: (1) The ship travels along a fixed parallel of latitude; (2) the ship travels along a small circle with constant speed; (3) the ship travels a small semicircle, after which it resumes a straightline course (along a large circle). In each case the author investigates the stability of the gyrocompass' motion.

T. Leser, USA

**2875. Pirogov, I. Z., On the stability of a gyroscopic system,** *Appl. Math. Mech.* (*Prikl. Mat. Mekh.*) **23**, 6, 1623–1626, 1959. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

The author applies the second method of Lyapunov to the problem of the stability of motion of a gyroscopic system. The differential equations describing the system involve two time-dependent coefficients. Difficulties in constructing a Lyapunov function are avoided by employing transformed variables for this quadratic form. Following classical procedure sufficient conditions for the asymptotic stability of the system are given in terms of the signs of determinant minors of the discriminant of the time derivative of the Lyapunov function. For a numerical case the region of asymptotic stability in the plane of the time-varying coefficients is compared with the region of stability obtained from the Hurwitz criterion. Good agreement is obtained.

R. Oldenburger, USA

## Elasticity

(See also Revs. 2831, 2849, 2912, 2913, 2914, 2915, 2916 2926, 2932, 2933, 2934, 2946, 2952, 2953, 2959, 2963, 2964, 3010, 3012, 3042, 3050, 3068, 3376)

**2876. Sedov, L. I., On the concepts of simple loading and on possible deformation paths,** *Appl. Math. Mech.* (*Prikl. Mat. Mekh.*) **23**, 2, 568–571, 1959. (Pergamon Press, Inc., 122 E. 55th St., New York 22, N. Y.)

Author studied the phenomenon of the deformation of a continuous medium under simple loading from the geometrical point of view apart from the physical nature of that medium. An exact analytical procedure demonstrates that the process of deformation in simply loaded state corresponds only to deformation of some special type for the entire body as a whole, from which it follows that, apart from the physical nature of the body, the concept of simple loading for arbitrarily distributed external surface forces has no meaning for finite deformations.

Starting from the geometrical compatibility condition which requires the Riemann tensor to vanish since the quadratic forms of the squares of the elements of length determine the element of length in the Euclidean space, author obtained two systems of equations for the components of finite tensor. One of them coincides with the system of Saint-Venant equations for the infinitesimally small strain, and the other can be considered as additional equations limiting the form of the strains, giving the strain which allows a simple loading.

Thus the concept of simple loading can only be applied within the framework of the approximate linear theory, which is valid only for geometrically small deformations.

T. Kanazawa, Japan

**2877. Liu, H.-C., The elastic stresses and deformations produced in a semi-infinite solid by tangential traction on its free surface** (in Chinese), *Acta Mech. Sinica* **3**, 2, 120–143, Apr. 1959.

Author solves the subject problem by using Hankel transform for cases where the distribution of surface tangential traction is axisymmetrical. The work is the extension of Terezawa's solution to elastic half-space under surface shear. General expressions for the solutions are given in integral forms.

Three examples are presented, in which the distributions of surface forces are such that the solutions can be evaluated in closed forms, with the first one numerically and graphically illustrated. Author hinted that the work is preliminary to the solution of certain thermal stress problems in half-space.

K. N. Tong, USA

**2878. Morgenstern, D., Mathematical foundations of plane stress (two-dimensional elasticity)** (in German), *Arch. Rational Mech. Anal.* **3**, 1, 91–96, Mar. 1959.

The approximate character of plane stress solutions in the theory of elasticity is considered. It is proved that the mean displacement converges in the mean to the solution of the plane stress problem as the thickness of the plate diminishes. This is a significant paper on the fundamental concepts of elasticity. It explains the nature of plane stress solutions used in many applications.

E. H. Dill, USA

**2879. Teodorescu, P. P., On the problem of the quarter of elastic plane** (in French), *C. R. Acad. Sci. Paris* **250**, 21, 3446–3448, May 1960.

**2880. Bauwens, J.-C., Homes, G. A., and Pankowski-Fern, Regina, On the initial stage of deformation of homogeneous and isotropic materials under simple stress** (in French), *C. R. Acad. Sci. Paris* **250**, 2, 290–292, Jan. 1960.

**2881. Narodetskii, M. Z., A problem in the theory of plane elasticity, solvable in a closed form** (in Russian), *Soobshch. Akad. Nauk GruzSSR* **19**, 3, 263–266, 1957; *Ref. Zh. Mekh.* no. 9, 1959, Rev. 10484.

This is an investigation of an elastic medium $S$, occupied by an infinite plate weakened by two openings of circular outline ($R$, $r$ are the radii of the openings, $R > r$). An evenly distributed load is applied to the outlines $L_1$ and $L_2$ with an intensity corresponding to $p$ and $q$ respectively. Suppose $a$ and $b$ are the distances from the beginning of the coordinates to the centers of the periphery. By using the method in the theory of the function of a complex variable the author obtains the following expression for the function of stresses $\varphi(z)$ and $\psi(z)$:

$$\varphi(z) = A(\alpha + \xi\mu'^2\beta) \quad \left(A = \frac{c\mu}{1-\mu^2\mu'^2}\right)$$

$$\psi(z) = -A\left[(1-c^{-2}\xi^{-2})(\mu^{-2}\alpha + \xi\beta) + \mu^{-2}\left(1-\mu\frac{a}{R}\right)\alpha^2 + \xi\left(1-\mu'\frac{b}{r}\right)\beta^2\right]$$

where

$$\alpha = \frac{R\mu}{R\mu + (z-a)}, \qquad \mu = \frac{R}{(a+b) - r\mu'} < 1, \qquad \xi = \frac{R}{r\mu\mu'}$$

$$\beta = \frac{r\mu'}{r\mu' - (z+b)}, \qquad \mu' = \frac{r}{(a+b) - R\mu} < 1, \qquad c = \frac{r}{R}$$

Continuing, author finds the value of $\sigma_\theta$ in points of the contours $L_1$ and $L_2$, by selecting in a determined way systems of polar co-

ordinates on the basis of the formula $\sigma_r' + \sigma_\theta = 4 \, Re \, \varphi'(z)$. A curve is also drawn for $\sigma_\theta$ with points lying on $L_1$ and $L_2$ with $c = \sqrt{2/2}$ and $R/(a+b) = 1/2$. A. K. Rukhadze

*Courtesy Referativnyi Zhurnal, USSR*

**2882. Valov, G. M., A problem regarding the deformation of an elastic round cylinder** (in Russian), *Trudi Sibirsk. Metallurg. In-ta*, no. 4/A, (*Prikl. Mat. Mekh.*), 29–32, 1957; *Ref. Zh. Mekh.* no. 9, 1959, Rev. 10469.

A solution is offered for the mixed axisymmetrical problem in the theory of elasticity for a round cylinder, symmetrically loaded relative to the central transverse section. On the lateral surface free from tangential stresses normal stresses are assigned; on the ends radical displacements and normal stresses are assigned. The loads on the ends and the displacements are resolved in Fourier series in accordance with Bessel functions of the first order. An error is to be found in formula (10) when determining the value of the arbitrary constant $a_1$. The correct value for this constant is $a_1 = f_0/G$. N. T. Glazunov

*Courtesy Referativnyi Zhurnal, USSR*

**2883. Brock, J. S., The relation between stress concentration and boundary displacement for simple openings,** *ASME Trans.* **82**E (J. Appl. Mech.), 2, 356–357 (Brief Notes), June 1960.

In this note an interesting relation between stress concentration and boundary displacement is pointed out which applies for openings in the form of circles and ellipses loaded parallel to a semiaxis. From author's summary

**2884. Legendre, R., Stress distribution in a half-plane** (in French), *C. R. Acad. Sci. Paris* **250**, 18, 2995–2996, May 1960.

**2885. Trotsenko, V. N., The stressed condition of a rock massive which has been weakened by an unstrutted shaft of elliptical transverse section** (in Russian), *Trudi Dal'nevost. Politekhn. In-ta* **49**, 1, 1–13, 1958; *Ref. Zh. Mekh.* no. 9, 1959, Rev. 10696.

The problem regarding the elastic equilibrium of an infinite plate weakened by an elliptical opening is solved in elliptical coordinates. The plate is compressed to infinity by forces acting in two mutually perpendicular directions. The stressed state of the plate, in its application to rock pressure, is investigated on the basis of A. H. Dinnik's known formula [Collect. Trudi Commn. for Management by Rock Pressure, Izd-vo AN.SSSR, 1938].

Note by the abstractor: The author's tribute to I. V. Rodin for work done on this question is misplaced. A solution of the analogous problem by the method of functions of a complex variable is given in N. I. Muskhelishvili's book ["Some basic problems in the mathematical theory of elasticity," Izd.vo AN.SSSR, 1933]. A solution in elliptical coordinates is also available in S. P. Timoshenko's book ["Theory of elasticity," ONTI, 1937]. The author makes no reference to these publications in his paper.

M. P. Sheremet'ev

*Courtesy Referativnyi Zhurnal, USSR*

**2886. Ku, T.-C., Stress concentration in a rotating disk with a central hole and two additional symmetrically located holes,** *ASME Trans.* **82**E (*J. Appl. Mech.*), 2, 359–360 (Brief Notes), June 1960.

**2887. Dutt, S. B., Stress concentrations around a small inclusion (formed by the revolution of a cardioid about its axis) on the axis of a circular cylinder under torsion,** *Bull. Calcutta Math. Soc.* **50**, 1, 29–33, Mar. 1958.

Author cites a series of known stress derivations for flaws and inclusions and adds one for the cardioid-profile inclusion on the axis of a shaft in torsion, using zero displacement for the surface of the inclusion. There are no numerical examples, graphs, or discussion. Z. W. Dybczak, Canada

**2888. Karnozhitskii, V. P., Stresses in long three-layered cylindrical shells where the temperature varies through the thickness** (in Russian), *Inzhener. Sbornik Akad. Nauk SSSR* **28**, 197–203, 1960.

Solution for plane axisymmetric thermal stress problem of elastic shell incased between elastic membranes is obtained by matching known elementary solutions. Temperature in shell is assumed to vary linearly; temperature in each membrane is taken as constant. Author makes no reference to modern literature on subject.

B. S. Wilson, USA

**2889. Knoschke, A., Elastic circular plates subject to the influence of a temperature on one side** (in German), ZAMM 40, 1/3, 40–46, Jan./Mar. 1960.

Paper explores the distribution of temperature and elastic stress and strain in a thick circular plate with a given temperature distribution on one surface and heat loss to the air through the other surface. The curved lateral surface is insulated.

L. Foppl, Germany

**2890. Adams, L. H., and Waxler, R. M., Temperature-induced stresses in solids of elementary shape,** *Nat. Bur. Stands. Monogr.* 2, 27 pp., June 1960.

This paper is for the use of practical engineers in estimating thermal stress components. Tables are given, from which the components can be found by addition and multiplication only, for slabs, cylinders and apheres whose surfaces are subject to a change of temperature which is either sudden or increasing at a linear rate. D. R. Bland, England

**2891. Nowinski, J., Transient thermoelastic problem for an infinite medium with a spherical cavity exhibiting temperature dependent properties,** Univ. Texas, Coll. Engng. (Contract DA-11-022-ORD-2059), 47 pp. + figs., 1960.

This paper is concerned with a polarly symmetric transient thermoelastic problem for an infinite medium with a spherical cavity, the boundary of the cavity being subjected to a sudden temperature rise. Thermal and elastic properties of the medium are assumed to be temperature-dependent. Using a perturbation method, general equations for the displacements and stresses corresponding to particular boundary-value problems have been found. An illustrative example, involving linear variation of conductivity and thermal expansion as well as quadratic variation of shear modulus with temperature is discussed in detail.

From author's summary by J. H. Weiner, USA

**2892. Sutherland, R. D., and Manville, S. M., Thermal stresses in a perforated square plate,** AFOSR TN 60-840 (Convair, Pomona, Calif., Rep. TM 349-19), 36 pp., June 1960.

The thermal stress problem in a square plate containing a central circular hole is solved using the complex analysis of Muskhelishvili and conformal mapping. A temperature distribution for this configuration is observed experimentally. This distribution is used in conjunction with the results of the analysis to calculate the thermal stresses in the plate. The stresses thus obtained are presented as functions of radial and angular displacement throughout the plate. From authors' summary

**2893. Shugalov, A. I., Temperature stresses in a block lying on a rigid foundation** (in Russian), *Trudi Leningrad Politekhn. In-ta* no. 196, 87–107, 1958; *Ref. Zh. Mekh.* no. 9, 1959, Rev. 10561.

The distribution of stresses in the wall due to concentrated forces applied to its face is the method used for the investigation of the temperature stresses in a rectangular wall clamped along the lower edge, the conditions prevailing being those of a one-dimensional thermal field. Numerical examples are given of the calculations for the thermal stresses in plates, the temperatures of which

varied with the height of the plates. The solution for the problem is obtained on the basis of the investigation of a semi-infinite strip with portions cut away from its length equal in length to the plate being examined; the tangential stresses acting on the cut-off face are disregarded; because of this the author's method can only be applied to low or drawn-out plates.

S. V. Aleksandrovskii
*Courtesy Referativnyi Zhurnal, USSR*

**2894. Adkins, J. E., Symmetry relations for orthotropic and transversely isotropic materials** (in English), *Arch. Rational Mech. Anal.* **4**, 3, 193–213, Jan. 1960.

This paper concerns the determination of a matrix polynomial suitably restricted to describe any symmetry properties of the material for transversely isotropic and orthotropic bodies in the general case involving kinematic matrices. For the transversely isotropic case, an alternate formulation is given based on the decomposition of the $3 \times 3$ symmetric kinematic matrices into a system of symmetrical and unsymmetrical $2 \times 2$ matrices.

By an extension of the symbolic method of the classical invariant theory, the number of coefficients and invariants necessary for the general representation of a transversely isotropic body is considerably reduced. The stress-strain relations for an elastic body are presented in two formulations.

Finally paper gives a generalization for $n+1$ matrices for orthotropic bodies.

A. L. Petre, Roumania

**2895. Adkins, J. E., Further symmetry relations for transversely isotropic materials** (in English), *Arch. Rational Mech. Anal.* **5**, 3, 263–274, 1960.

Stress-deformation relations are investigated for the case where the stress depends upon strain and strain-rate tensors and also upon certain vector fields. The necessary forms appropriate to symmetries associated with an axis are determined using the theory of invariants. Results may be of value to those seeking formulation of material laws of stress and strain in the presence of temperature gradients or electromagnetic fields.

Earlier work is reported in Adkins, J. E., *Phil. Trans. Roy. Soc.* (*A*) **250**, p. 519, 1958; AMR **12**(1959), Rev. 4526; *Arch. Rational Mech. Anal.* **4**, p. 193, 1960; see preceding review.

W. S. Hemp, USA

**2896. Glazunova, N. T., A solution for the first basic problem in the theory of elasticity for an orthotropic wedge** (in Russian), *Izv. Vyssh. Uchebn. Zavedenii. Str-vo i Arkhitekt.* no. 5, 11–21, 1958; *Ref. Zh. Mekh.* no. 9, 1959, Rev. 10485.

An investigation is carried out for the plane problem on the elastic equilibrium of an orthotropic wedge-shaped cantilever loaded with force and moment applied to the apex of the wedge, and with an arbitrary normal and tangential load distributed on the sides. The method of successive approximations is proposed for the solution, based on the utilization of the equations for the plane problem in stresses, two equations for equilibrium and an equation obtained from the condition of mutual deformation. A system of "initial" stresses $\delta_{y,0}$, $\delta_{z,0}$, $\tau_{y,z,0}$ (which, in particular, are taken to be equal to zero) is assigned, and in accordance with them the first, second and following approximations are found successively from the equations of the plane problem and the conditions prevailing on the sides and in the transverse sections, used in the determined order. In the general case of loading the process of finding the successive approximations leads to a precise solution only on the boundary. For the load distributed on the sides according to the principle for a whole polynome a precise solution is arrived at by means of a finite number of approximations; some cases of this nature are investigated.

S. G. Lekhnitskii
*Courtesy Referativnyi Zhurnal, USSR*

**2897. Mitra, M., On the solution of problems of dynamic plane elasticity for anisotropic media,** *Quart. J. Mech. Appl. Math.* **13**, 3, 369–373, Aug. 1960.

It is shown how the equations of elasticity, pertaining to a dynamic state of plane stress in an anisotropic plate (in which, however, the time appears only in the combination $x-ct$), may be solved for the components of stress and displacement expressed in terms of a pair of analytic functions of each of two different complex variables. The resulting expressions are then applied to evaluating the field variables in the problems of a moving parabolic punch and a moving dislocation. It should be noted that the general solution is subject to a rather complicated set of conditions on the velocity, $c$, of the disturbance.

H. Deresiewicz, Italy

**2898. Vacca, Maria T., Forces and deformations in an elastic sphere with transverse rigidity that varies with the radius and with torsional surface loading** (in Italian), *Atti Accad. Sci., Torino* **94**, 54–66, 1959/60.

The stress and displacement are determined within an elastic spherical region made up of an internal concentric sphere of radius $R_1$, with constant rigidity, and a concentric spherical shell extending from $R_1$ to $R_2$ in which the rigidity varies with the radius. Surface tractions are applied at $R_2$ and are confined to tangential torsional forces. Method of solution is to reduce the problem to that of determining the displacement within the sphere. This is finally accomplished by the separation of variables in a straightforward manner. The separated equations are shown to yield a solution which is here expressed in terms of an infinite series involving Ferrer functions which are related to Legendre polynomials and their derivatives. A good portion of the paper is devoted to determining the coefficients of this series, so that the necessary properties of the Ferrer functions are included in the discussion.

Two particular cases are considered; one, in which $\mu = \mu_0 r^k$ and $R_1 = 0$, and the other in which $\mu = \mu_0 \exp(\frac{1}{2}\beta r^2)$ and $R_1 > 0$ with $\beta^2 \ll 1$.

In general the paper is clearly written and thus easily followed. It is little trouble for the reader to correct the vector equation on p. 61, for example.

W. C. Orthwein, USA

**2899. Galfayan, P. O., and Chobanyan, K. S., Approximate solution of some problems of torsion of shafts with a thin stiffening covering** (in Russian), *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk* no. 4, 85–92, July/Aug. 1959.

The solution is based on the Saint-Venant torsional theory. It is assumed that the stress function varies linearly through the thickness of the stiffening covering. Using elliptical or bipolar coordinates author solves the torsion problem for a solid elliptical cylinder, for a hollow cylinder whose cross section is bounded by confocal ellipses and for a hollow beam whose cross section is the region bounded by two eccentric circles. The stiffening covering in this case is assumed to be of uniform thickness.

C. Hoschl, Czechoslovakia

**2900. Goldsmith, W., and Lyman, P. T., The penetration of hard-steel spheres into plane metal surfaces,** *ASME Trans.* **82**E (*J. Appl. Mech.*), 4, 717–724, Dec. 1960.

An experimental investigation was undertaken to determine the force-indentation relations governing the contact of hard-steel balls and plane surfaces of various metals both under static and dynamic conditions, the latter involving the Hopkinson-bar technique, with maximum elastic strain rates of 500 $sec^{-1}$. Excellent correlation was obtained between the measured permanent crater diameter at the contact point and that calculated from strain-gage data by means of an equation treating the bar as a one-dimensional member. A comparison also was effected between the static and dynamic force-indentation curves, the Hertz law of contact, and a

relation based upon the concept of a constant flow pressure in the plastic regime. From authors' summary

**2901. Gazis, D. C., Graphical investigation of geometric aspects of the Hertz problem,** *ASME Trans.* **82**E (*J. Appl. Mech.*), 4, 735–737 (Brief Notes), Dec. 1960.

**2902. Solomon, L., Some remarks on the problem of the elastic contact** (in French), *C. R. Acad. Sci. Paris* **250**, 17, 2843–2845, Apr. 1960.

**2903. Bramble, J. H., and Payne, L. E., An analogue of the spherical harmonics for the equations of elasticity,** AFOSR 6 (Univ. Maryland, Inst. Fluid Dynam. Appl. Math. TN BN-222), 16 pp., Nov. 1960.

In this paper authors obtain for the interior of an elastic sphere a set of solutions of the displacement equations of classical elasticity which form a complete orthogonal system of vectors on the surface of the sphere. These functions should prove useful in approximating solutions of boundary-value problems in elasticity.

From authors' summary

**2904. Rivaud, J., On circular flexure in the theory of finite deformations** (in French), *C. R. Acad. Sci. Paris* **251**, 1, 32–34, July 1960.

**2905. Giet, A., On the elastic behaviour of materials** (in French), *C. R. Acad. Sci. Paris* **251**, 3, 327–329, July 1960.

## Viscoelasticity

(See also Revs. 2895, 3011, 3017, 3018, 3035, 3072)

**2906. Bessonov, M. I., and Kuvshinskii, E. V., Determining the creep of hard polymers,** *Indust. Lab.* **25**, 9, 1171–1174, July 1960. (Translation of *Zavod. Lab., SSSR* **25**, 9, 1117–1120, Sept. 1959 by Instrument Society of America, Pittsburgh 22, Pa.)

Authors' approach is to apply standard apparatus used for creep testing in general for the testing of hard polymers. Paper describes a simple device with horizontal configuration which can apply floating weight, weight seated on the stationary eccentric axis, lever with unloading by steps, or counterweight. Authors used pulleys with counterweights and a contoured eccentric loading device. The deformation of the sample was registered by an electric slide wire transducer. Reviewer believes that this apparatus does not permit one to maintain constant stress conditions with precision, because the end section of the samples is deformed as well as the working part.

L. J. Bonis, USA

**2907. Kachanov, L. M., Approximate solution of problems of steady creep** (in Russian), *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk* no. 3, 84–95, May/June 1959.

In author's opinion the mathematical difficulties involved in solving problems of creep by using a nonlinear theory of creep justify the use of more simplified theories; they may also give results closer to the exact solution to the problem.

In this work the relationship between rate of deformation ($H$) and applied stress ($T$) is assumed to be of the form $H = BT^m$. Author discusses at length the solutions for limiting values of the parameter $m$, namely, $m = 1$ and $m \to \infty$. He concludes that, with increasing value of $m$, the stress conditions lead to the ideally plastic state, providing that the given problem permits totally plastic conditions.

An approximate method of solution is presented: it is based on equation

$$\int_V \bar{T}^{m+1}\, dv = \min$$

where $V$ is volume, $\bar{T} = T/T_1$, and where $T_1$ is a particular value of stress. To illustrate the proposed method four problems are discussed:

1. Creep of a hollow sphere subjected to the internal pressure.
2. Creep in pure bending of a beam of rectangular cross section.
3. Creep in tortion of a beam of square cross section.
4. Creep of a plate bent in the axes of symmetry.

W. D. Sylwestrowicz, USA

**2908. Patel, S. A., and Venkatraman, B., Creep bending of annular plates,** AFOSR TN 60-528 (Polyt. Inst. Brooklyn, Dept. Aero. Engng. Appl. Mech., PIBAL Rep. 559), 19 pp. + figs., May 1960.

The present report is concerned with the creep bending of annular plates subjected to pure bending moments. Annular plates on simple and fixed supports are considered, and the analyses are carried out on the bases of the elastic analog and a creep flow law formulated in similar terms to those of the plastic flow laws. Solutions for moments and deflections are presented.

From authors' summary

**2909. Rossiiskii, V. A., Influence of creep and change of the modulus of deformation of the concrete on the conditions of stability for concrete and ferroconcrete arches** (in Russian), *Trudi Khar'kovsk. Avtomob.-Dor. In-ta* no. 21, 99–112, 1958; *Ref. Zh. Mekh.* no. 9, 1959, Rev. 10884.

An investigation is made of the creep in an evenly loaded hinge-supported ferroconcrete arch. The axis of the arch is so fixed that in any section only the normal force would be acting. A calculation is made for the change with time of the modulus of deformation. Calculations are also made for the auxiliary thrust which is produced as the result of creep, and for the critical load.

V. S. Namestnikov
*Courtesy Referativnyi Zhurnal, USSR*

**2910. Biot, M. A., The influence of gravity on the folding of a layered viscoelastic medium under compression,** *J. Franklin Inst.* **267**, 3, 211–228, Mar. 1959.

With geological applications in mind, author treats problem of folding of a layer of viscoelastic material either lying above one layer, or embedded horizontally between two layers, of other viscoelastic media under a combined state of prestress; prestress is due to gravity and to horizontal compression in the layer. Simplifying assumptions are made so that a "plate-type" equation may be used for the layers. Dominant wavelength and relative amplitude of folding is obtained to yield indication of magnitude of instability.

For a layer embedded between two viscoelastic media it is shown that, if underlying material is denser than that above, gravity has stabilizing effect. If underlying material is less dense than that above, layer is unstable.

B. Bernstein, USA

**2911. Vitovec, F. H., Effect of relaxation on the behavior of materials under combined alternating and static stress,** *ASME Trans.* **82**D (*J. Basic Engng.*), 2, 441–446, June 1960.

The effect of temperature on the stress range diagram is discussed and the particular characteristics of the curves for unnotched and notched specimens are analyzed. Excluding metallurgical factors from consideration, it is suggested that relaxation is the principal mechanism which influences the behavior of polycrystalline metals under combined alternating and static stress.

Two parallel nonlinear Maxwell units are used to represent the relaxation mechanism at elevated temperature. An analysis of this model shows that relaxation occurs to an asymptotic finite value of stress which is a function of the initial stress. The same model is applied for representing the behavior in the stress range diagram with the assumption that a linear relationship exists at low temperature between the fatigue strength and the normal stress acting on the planes of reversed slip.

From author's summary

**2912. Landau, H. G., Weiner, J. H., and Zwicky, E. E., Jr., Thermal stress in a viscoelastic-plastic plate with temperature dependent yield stress,** *ASME Trans.* **82** E (*J. Appl. Mech.*), 2, 297–302, June 1960.

Equations are given for the determination of transient and residual stresses in plates subject to transient temperature distributions, based on the assumption of a viscoelastic, perfectly plastic material obeying a von Mises temperature-dependent yield condition. A numerical procedure for integrating the equations is developed and applied to the case of a symmetrically cooled plate. It is found that, for steel, viscoelasticity has little effect on the residual stress distribution, but the temperature dependence of yield stress is important. The types of residual stress distribution after cooling are similar to those for an elastic-plastic material with constant yield stress, and for this case the residual stress is given approximately by formulas developed earlier for a slowly varying heat input.

From authors' summary by M. Holt, USA

## Plasticity

(See also Revs. 2895, 2959, 3002, 3003, 3057, 3370, 3372)

**2913. Perlin, P. I., Approximate methods of analysis of elasto-plastic problems** (in Russian), *Inzhener. Sbornik Akad. Nauk SSSR* **28**, 145–150, 1960.

Using analytical extension of certain functions developed by Kolosov-Muskhelishvili, approximate method is given to find stresses at infinity and at boundary points between elastic and plastic zones when two special boundary points are initially given in an infinite plate with a hole of any symmetrical shape, the plate being either in a plane stress or plane strain state. Plastic conditions of Saint-Venant and von Mises are used. Two possible cases are treated: when plastic zone is all around the hole and when it cuts it. In first case, when the extension of functions has no odd points, an approximation is used to define these functions in elasticity with given boundary condition and developing the function into a Fourier series. To find the coefficients of the series, the boundary points are taken to be on a pencil of rays through the hole center plus two given points. The correct points are obtained by equating corresponding stress components obtained for the same points in elasticity and in plasticity. These equations are satisfied by variation of the positions of points.

In second case, by the extension of functions up to the hole the auxiliary problem in elasticity of a plate with a hole and loaded along two parts of arc with new boundary condition is to be solved.

The basic question is the possibility of the analytical extension of functions in the given zone. Reviewer believes that this paper is a nice contribution to the old known problem of finding the boundary between elastic and plastic zones in a plate with a hole and loaded at infinity. B. Kuzmanovic, Sudan

**2914. Tsurkov, I. S., Elasto-plastic deformations in thin-walled cylinders near a rigid ring** (in Russian), *Inzhener. Sbornik Akad. Nauk SSSR* **28**, 182–189, 1960.

A long circular cylindrical shell with a rigidly fixed ring around its central portion is subjected to uniform internal pressure. The small elastic-plastic deformation is described by a fourth-order beam-type equation for deflection with right-hand members involving it again nonlinearly. A first approximation to the solution is now obtained by using the elastic solution, given by Lurie, in the right hand members. The deflection, curvature, etc. of this solution are shown against pressure for various values of a constant depending on the dimensions and moduli of the materials.

G. A. Nariboli, India

**2915. Gerasimov, I. S., Axisymmetric elasto-plastic deformations of closed circular cylindrical shells** (in Russian), *Inzhener. Sbornik Akad. Nauk SSSR* **28**, 241–246, 1960.

A long cylindrical shell has a plane bottom connected through a ring. The elastic solution for given internal and external pressures is determined by continuity requirements, the bottom deflection being governed by usual plate equation and that of the shell by a beam-type fourth-order equation. The bottom is then assumed to go into plastic state. Its deflection is taken in a similar form with an undetermined constant determined by minimizing the work of deformation. Assuming small deformation the stress intensity is taken to be a cubic function of strain intensity.

G. A. Nariboli, India

**2916. Conway, H. D., Elastic-plastic bending of curved bars of constant and variable thickness,** *ASME Trans.* **82**E (*J. Appl. Mech.*), 4, 733–734 (Design Data and Methods), Dec. 1960.

**2917. Ershov, L. V., The elasto-plastic state of an eccentric shrunk onto an elastic shaft** (in Russian), *Vestn. Mosk. In-ta* no. 5, 13–16, 1957; *Ref. Zh. Mekh.* no. 9, 1959, Rev. 10607.

The elasto-plastic stress and the deformed state of an eccentric which has been drawn onto an elastic shaft at an assigned tension are investigated. The solution is given in the form of a series. An assumption is made that on the boundary of the contact along the periphery the pressure is constant. This assumption is only valid when the eccentricities are not large. A numerical example is given, curves are drawn to show the changes of the peripheral and radial stresses along the radius for the most dangerous section of the eccentric. No evaluation is given for the accuracy of the obtained solution. N. D. Tarabasov

*Courtesy Referativnyi Zhurnal, USSR*

**2918. Moguchii, L. N., Treatment by pressure of light metals and alloys possessing small plasticity** (in Russian), Light alloys, Vol. I, Moscow, 1958, 423–438; *Ref. Zh. Mekh.* no. 9, 1959, Rev. 10636.

Results are given of an investigation on the influence of an (iron) ring on the deformability of light alloys, possessing small plasticity, in its application to forging processes. The study includes the determination of the hydrostatic pressure on the test sample created by the (iron) ring, the investigation of the deformability of alloys of low plasticity when experimenting with various types of rings, and clarification of the character of the flow of the material when it sags in the rings and when it sags without them. A deduction is put forward showing the relation of the pressure of the ring on the test sample to the degree of deformation. Another deduction is furnished, on the basis of the investigation on the influence of the ring on the deformability of the blank when they sag jointly, to the effect that the use of rings made from a plastic and sufficiently tough material (duralumin rings) increases the tendency of the sample to deform and opens up the possibility to carry out the sagging with any practically essential degree of forging. A conclusion is reached, based on the investigations of the distribution of deformations when sagging with or without the rings is taking place, regarding the presence of tension stresses

in the radial direction of the forging, and also regarding the extremely high degree of irregularity in the deformations.

Yu.G. Maksimov
*Courtesy Referativnyi Zhurnal, USSR*

**2919. Geogdzhaev, V. O., Compressed and drawn orthotropic plastic strips** (in Russian), *Inzhener. Sbornik Akad. Nauk SSSR* **29**, 80–91, 1960.

Author discusses the stresses in orthogonal anisotropic strips in plain strain plastic problems such as compression of a semi-finite and finite strip, drawing of strip through a die and rolling of it between two rolls. In cases of compression between two rigid plates and the drawing the maximum developed friction is taken into consideration, but in rolling no friction is assumed to exist between rolls and strip.

Obtained formulas for stress components are given in close relation to those given by V. V. Sokolowski for isotropic bodies.

In each case treated, drawings of characteristics and stresses are produced.

The obtained results are of interest for practical engineering problems for bodies of three mutually perpendicular axes of anisotropy.

B. Kuzmanovic, Sudan

**2920. Prager, W., and Shield, R. T., Minimum weight design of circular plates under arbitrary loading** (in English), *ZAMP* **10**, 4, 421–426, July 1959.

Plate considered is of sandwich type with a core of constant thickness between two identical thin face sheets of *variable* thickness $h(r, \theta)$. Material of the sheets is elastic-perfectly plastic and follows Tresca's yield condition. Problem is to find $h(r, \theta)$ so that the plate can carry the given load under the requirement of minimum weight design [Drucker and Shield: Bounds on minimum weight design, *Quart. Appl. Math.* **15**, 269–281, 1957; AMR **11**(1958), Rev. 2125]. For circular plates (simply supported or clamped at the edge) the deformation modes which satisfy the said requirement are rotationally symmetric.

For both conditions of support authors derive from the moment equilibrium equation a partial differential equation for $M_0 = K.h$ (maximum bending moment). They indicate the solution for a concentrated load $P$ at an arbitrary point, and integrate this solution in order to obtain that for a distributed load $p(r, \theta)$.

Work is an extension of Onat, Schumann and Shield: Design of circular plates for minimum weight, *ZAMP* **8**, 485–499, 1957 [AMR **12**(1959), Rev. 4919].

G. H. Beguin, Switzerland

**2921. Kliushnikov, V. D., On a possible manner of establishing the plasticity relations,** *Appl. Math. Mekh.* (*Prikl. Mat. Mekh.*) **23**, 2, 405–418, 1959. (Pergamon Press, Inc., 122 E. 55th St., New York 22, N. Y.)

The principle of independent response in plasticity calls for certain known criticism. Therefore it is natural to establish a theory of plasticity which includes the concept of angular point without application of above-mentioned principle in any form. Such an attempt had been made by author previously for the problem of plane loading paths [see *Prikl. Mat. Mekh.* **22**, no. 1, 1958; AMR **12**(1959), Rev. 3798]. However, one of the assumptions introduced, namely the plasticity conditions for radial loading, is not quite satisfactory. Thus in this paper, author introduces a more general one in place of it and delineates precisely all the assumptions introduced and formulates them in such a manner that following investigations bear a geometric character.

New system of assumptions leads to improved results. Author obtained two different expressions for the increment of plastic deformation corresponding to the direction of additional loading. Results of calculation based on them are provided in case of simple loading, which discloses possibility of satisfing the deformational theory (within certain limits) without violating basic assumptions of mechanics of solid media.

Most essential conclusion is the fact that for paths which deviate rather strongly from simple loading a conformance with relationships of the theory of Hencky-Nadai is obtained.

T. Kanazawa, Japan

**2922. Payne, H., and Czyzak, S. J., On the torsion of a thin-wall cylinder following a plastic extension,** *J. Mech. Phys. Solids* **8**, 1, 39–44, Jan. 1960.

H. Payne [*J. Mech. Phys. Solids* **7**, p. 126, 1959] extended G. I. Taylor's [*J. Inst. Metals* **62**, p. 357, 1938] model for calculating the stress-strain relation for a crystalline aggregate in the plastic region following a linear strain path to include the complete stress-strain relation for a nonlinear strain path. To test this theory, it was applied to the case of a thin-walled cylinder subjected to an incremental torsional strain following an existing tensile or compressive plastic strain.

Experimental results came into approximate agreement with the theory after the application of a torsional stress in excess of approximately one-fourth the tensile stress in effect prior to the application of the torsion strain. This indicates that the new model requires a much larger ratio of the plastic to the total strain.

D. Kececioglu, USA

**2923. Zhukov, A. M., Properties of D16T alloy subject to tension combined with torsion** (in Russian), *Inzhener. Sbornik Akad. Nauk SSSR* **29**, 55–62, 1960.

The present study is a continuation of author's publications in *Izv. Akad. Nauk*, June 1954 and Sept. 1957. D16T is identical to the aluminum alloy 14 S-T6 about which Marin and Wiseman wrote in the *Journal of Metals* **5**, 9 (Sec. 2), p. 1181, 1953.

By testing thin-walled tubes under longitudinal stress and internal pressure author previously determined that D16T and MA2 cast in rods are anisotropical. He now combines tension and torsion on 12 specimens.

On the results, he applies conditions of plasticity following Huber-Mises and writes for the

older tests $$A11\sigma_1^2 + 2A12\sigma_1\sigma_2 + A22\sigma_2^2 = 1 \qquad [1]$$

new tests $$A11\sigma_1^2 + 2A14\sigma\tau + A44\tau^2 = 1, \qquad [2]$$

which formulas he transforms into curves. The results agree poorly with (1) and much better with (2).

Rupture approximately conforms to $\sigma^2/\sigma_r^2 + \tau^2/\tau_r^2 = 1$ in which $\sigma_r$ and $\tau_r$ determine rupture by pure tension or pure torsion.

N. B. Van Albada, Holland

**2924. Yagi, Yu. I., Shishmarev, O. A., Investigation of plastic deformation, with simultaneous tension and torsion, of thin-walled tubular metal samples** (in Russian), *Zavod. Lab.* **24**, 10, 1243–1245, 1958; *Ref. Zh. Mekh.* no. 9, 1959, Rev. 10968.

A description is given of the apparatus used for the testing of thin-walled tubes for simultaneous tension and torsion. The apparatus in question was used for the study of deformational aelotropy in thin-walled nickel tubes (diameter 5 mm and wall thickness 0.2 mm), preliminarily subjected to plastic tension (up to 4.5% of the residual relative elongation) or torsion (up to 1.7% of the relative shear). Experimental data are given for the determination of the surface of yield in coordinates $\sigma - \tau\sqrt{3}$ after preliminary tension beyond the limit of elasticity. It is shown that in this case the surfaces have the form of areas with centers on the axis $\sigma$, transposed towards the tension side.

I. A. Shubin
*Courtesy Referativnyi Zhurnal, USSR*

**2925. Conte, R., Dreyfus, B., and Weil, L., Plastic torsion of iron "whiskers" at temperature of 300–320°K** (in French), *C. R. Acad. Sci. Paris* **250**, 2, 337–339, Jan. 1960.

**2926. Rossard, C., and Blain, P., Transient phenomena caused by a sudden change of temperature during the plastic deformation of warm steel** (in French), *C. R. Acad. Sci. Paris* **250**, 3, 533-535, Jan. 1960.

## Rods, Beams and Strings

(See also Revs. 2844, 2899, 2972, 3046, 3050, 3052, 3057, 3285, 3369)

**2927. Quinlan, P. M., Elastic beam theory for the electronic computer—a recurrence approach,** AFOSR TN 60-936 (Contract AF61(514)-1163), 40 pp., June 1960.

A new comprehensive approach to elastic beam theory is presented, based on the use of generalized step functions. Uniform beam on continuous supports, whether rigid or elastic, is solved using a new formulation of the three-moment equation in conjunction with a shear-recurrence formula, called the "three-supports equation." An alternative solution for continuous uniform beams on elastic support is given using generalized step-functions.

Part two, dealing with nonuniform beams, is based entirely on recurrence formulas obtained from generalized step-function approximations to the deflection, and slope equations by obtained finite differences. Solutions are obtained for all loading conditions, including concentrated moments and end thrust, or end tension.

In all cases the solution is reduced to a number of simultaneous equations.

From author's summary by O. Halasz, Hungary

**2928. Engel, E., Stability of curved, continuous railway tracks** (in German), *Öst. Ing.-Arch.* **14**, 2, 139-152, June 1960.

Using energy methods, author deduces formulas for the computation of a stability criterion for curved, continuous railway tracks under the action of axial forces. He concludes that the risk for buckling of straight tracks is small, but is so large for curved continuous tracks with small radii of curvature that he questions the advisability of building such tracks.

F. I. Niordson, Denmark

**2929. Wise, S., Lindsay, D., and Duncan, I. G. T., The strength of rails with particular reference to rail joints,** *Proc. Instn. Mech. Engrs.* **174**, 9, 371-408, 1960.

This three-part paper describes recent work on rails carried out by the Research Department of British Railways.

Part I details the rail sections considered, theoretical considerations of longitudinal bending strength and an analysis of failures in service.

Part II describes a site investigation which was made into the dynamic stresses experienced by rail joints under traffic. Stresses in both flat bottom (F.B.) and bull head (B.H.) rail joints have been measured with both normal and abnormal conditions of sleeper packing. Electrical resistance strain gages were used at the calculated positions of maximum stress in each fishbolt hole, and at other positions in the joint.

The work initially covered the effects of steam locomotives and steam-hauled stock, but was later extended to a second site so that electric locomotives and multiple-unit electric stock could be similarly covered.

The work initially covered the effects of steam locomotives and steam-hauled stock, but was later extended to a second site so that electric locomotives and multiple-unit electric stock could be similarly covered.

The investigation brought out the marked influence of speed, wheel diameter and axle load on the magnitude of the stresses induced; and in view of the prospective increase in speeds, and the employment of diesel electric and electric locomotives with heavy axle loads on smaller wheels than as is usual currently, importance attaches to these results and their implications on the life expectancy of rail steel under fatigue loading.

Part III describes fatigue tests which have been made on B.H. and F.B. rails, both plain and when drilled with fishbolt holes. The susceptibility to cracks starting at bolt holes, due to shear stresses in the web, is compared for the two rail sections, and some consideration is given to various methods of increasing the resistance to cracking of this type. It is shown that ordinary atmospheric corrosion greatly reduces the fatigue strength of rails, and any proposed method of reducing bolt hole failures must therefore take this into account.

From authors' summary

**2930. Kuchinskii, A. F., Calculations for a round thin-walled beam with an open contour of the transverse section** (in Russian), *Izv. Vyssh. Uchebn. Zavedenii. Aviats. Tekhn.* no. 2, 53-63, 1958; *Ref. Zh. Mekh.* no. 9, 1959, Rev. 10784.

A method of calculation is described for an unclamped evenly supported round thin-walled beam on the basis of the theory of calculations of thin-walled constructions and the method of integration of a system of ordinary differential equations proposed by Yu. G. Odinakov [*Trudl Kazansk. Aviats. In-ta* 1946, (18); 1949, (23)]. The system of differential equations for equilibrium is recorded and also for the joint deformations. An expression is derived for the general integral of the system. The solution of the system is examined in the light of various assumptions relating to the roots of its characteristic equation. General expressions are given for the axial forces in the longitudinal bracing and the linear forces of shear in the panels of the sheating. The possibility is indicated of utilizing the solutions obtained for the calculations of thin-walled structures of the type of the fuselage of an aeroplane with an orifice.

M. S. Kornishin

*Courtesy Referativnyi Zhurnal, USSR*

**2931. Brungraber, R. J., Strength of welded aluminum-alloy box beams,** *Welding J.* **39**, 10, 417-423 (Res. Suppl.), Oct. 1960.

**2932. Ferrarese, G., Finite deformation of a tubular solid with curved generator** (in Italian), *Atti Accad. Naz. Lincei, R. C. Cl. Sci. Fis. Mat. Nat.* **27**, 6, 347-355, 1959.

Finite deformations of elastic rods having curvilinear generators are considered; results are more general than for rectilinear generators, considered in a paper yet to be published. Cubical dilatation is shown to be linear function of coordinates. Formulas obtained, when particularized for infinitesimal displacement, are shown to be identical with those of Love ("Elasticity," 4th. ed., page 392). Methods of differential geometry are used obscurely, and paper is tedious to read since most notations are undefined; there are no diagrams to assist the reader. Strain components alone are considered, and no application of the results obtained is discussed.

F. A. Gerard, Canada

**2933. Gudushauri, I. I., Analysis of foundation strips of finite rigidity taking account of reactive shear stresses** (in Russian), *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk* no. 4, 77-84, July/Aug. 1959.

Statically loaded foundation strip rests on an elastic isotropic half-space characterised by Young's modulus and Poisson's ratio. Author considers a plane strain case. It is assumed that the foundation strip obeys Bernoulli-Euler law, and the usual beam theory is used. The reactive normal and shear stresses are expressed by means of Chebyshev's polynomials. As an example a strip of finite length with uniform load is considered. The influence of reactive shear stresses on diminution of deflection and bending moment is briefly discussed and may be significant; but the distribution of normal reactive stresses is not excessively af-

fected, except the bending stiffness of foundation strip is relatively small. C. Hoschl, Czechoslovakia

**2934. Gregory, M., The bending and shortening effect of pure torque,** *Austral. J. Appl. Sci.* **11**, 2, 209–216, June 1960.

The method used by the author in a previous paper [AMR **14**(1961), Rev. 109] is generalized for thin-walled sections of any form. Experimental verification is obtained for large twists and three sections made from sheet brass. Good agreement was obtained. W. M. Shepherd, England

**2935. Ter-Mkrtich'yan, A. N., Deflection and torsion of thin and thin-walled beams loaded with longitudinal forces** (in Russian), *Trudi Tyl'sk. Mekhan. In-ta* no. 12, 209–226, 1958; *Ref. Zh. Mekh.* no. 9, 1959, Rev. 10783.

A system of three linear differential equations relative to the function of deflection in two planes and an angle of torsion is obtained for a rectilinear elastic beam of continuous section, being subjected to deflection, torsion and tension (or compression). The function of deplanation of the transverse sections entering these equations is not determined. On the basis of these equations, differential equations for the equilibrium of thin-walled beams of open or closed profile are obtained. Terms are contained in the equations which take into account the action of the torsional moment applied to the ends of the beam.

V. F. Lukovnikov
*Courtesy Referativnyi Zhurnal, USSR*

**2936. Minasyan, R. S., Torsion and flexure of aelotropic prismatic-shaped beams with a transverse section in the form of a parallelogram** (in Russian), *Izv. Akad. Nauk, ArmSSR, Ser. Fiz.-Matem. Nauk* **11**, 3, 41–62, 1958; *Ref. Zh. Mekh.* no. 9, 1959, Rev. 10498.

Problems are investigated on the torsion and flexure of a prismatically shaped beam possessing aelotropy of a special form: in every point of the beam a plane of elastic symmetry is to be found, each perpendicular to the axis. In the case of torsion the function for the stresses satisfies the differential equation

$$\frac{\partial^2 U}{\partial \xi^2} - 2\alpha \frac{\partial^2 U}{\partial \xi\, \partial \eta} + \frac{\partial^2 U}{\partial \eta^2} = -2$$

in which the variables do not divide. The author presents the solution of this equation simultaneously by means of two different series. The satisfying of all the boundary conditions leads to an infinite system of algebraical equations for the coefficients in these series. In cases of practical interest the systems obtained are found to be regular. A numerical example is given. A solution is derived in analogous fashion for the problem of flexure of an aelotropic beam. V. K. Prokopov
*Courtesy Referativnyi Zhurnal, USSR*

**2937. Solov'ev, Yu. I., The action of a concentrated force on an eccentric ring with a stiffened contour** (in Russian), *Trudi Novosib. In-ta Inzh. Zh.-d. Transp.* no. 14, 23–38, 1958; *Ref. Zh. Mekh.* no. 9, 1959, Rev. 10492.

By utilizing bipolar coordinates a solution is found for the problem of the elastic equilibrium of an eccentric ring, stiffened by a thin rim possessing rigidity against tension, on which a concentrated force is acting. It is assumed that this force is balanced by a certain assigned distribution of stresses on both the contours of the ring. A calculation for the ring is given when it is compressed by two equal forces acting along the diameter of the outer contour. In the calculation the first eight terms of the Fourier series are retained. M. P. Sheremet'ev
*Courtesy Referativnyi Zhurnal, USSR*

**2938. Gemma, A. E., Ring with edge moments,** *J. Aero space Sci.* **27**, 11, 867–868 (Readers' Forum), Nov. 1960.

# Plates, Shells and Membranes

(See also Revs. 2844, 2885, 2888, 2889, 2894, 2910, 2912, 2914, 2915, 2920, 3016, 3053, 3054, 3055, 3056, 3307)

**2939. Hieke, M., A consideration of the dynamics of circular membranes** (in German), *ZAMM* **39**, 12, 476–483, Dec. 1959.

The differential equation of motion governing the transverse deflection of a circular membrane is integrated by using Dirichlet's discontinuous factor. The forcing function consists of a uniform pressure acting over a segment of the circular area for time $t > 0$. By superposition some special cases are obtained. No numerical examples are given. Y.-Y. Yu, USA

**2940. Grzedzielski, A. L. M., Membrane theory of large spherical radomes,** Nat. Res. Counc. Canada, Aero. Rep. LR 278, 67 pp. + tables, May 1960.

The membrane analysis of incomplete thin spherical domes bounded by a parallel circle below the equator and subject to wind forces is considered. The first loading is considered to be symmetric with respect to the vertical diameter of the sphere. A solution is found in terms of Legendre polynomials for all stresses and displacements. A second loading, consisting of unit loads applied normally and tangentially to the spherical surface at given points on the surface, is also considered. A matrix solution of the membrane equations is established which is well-suited to digital computer applications. Both the solutions permit an elastic support at the foundation of the sphere. Examples indicate that both solutions involve only moderate computational efforts.

W. A. Nash, USA

**2941. Aggarwala, B. D., Bending of an isotropic triangular plate, two of whose edges are simply supported and the third clamped,** *ASME Trans.* **82** E (*J. Appl. Mech.*), 2, 357–358 (Brief Notes), June 1960.

**2942. Bailey, R., and Hicks, R., Behaviour of perforated plates under plane stress,** *J. Mech. Engng. Sci.* **2**, 2, 143–161, June 1960.

A rather complete and detailed theoretical and experimental study on the problem of infinitely perforated plates under plane stress is presented; a problem which has been attacked previously by several authors. Both cases of square systems and diagonal systems of holes are investigated. Graphs are given for the effective elastic constants in function of the ratio between pitch and radius of the holes. Similarly the stress distributions along principal sections are shown in diagrams with the foregoing ratio as parameter. The calculations were made with a computer using ordinary trigonometric series in polar coordinates. For this a finite number of terms were taken as an approximation, so that the periodicity conditions were fullfilled only at discrete points on the lines of symmetry, whereas the boundary conditions at the edges of the holes were satisfied completely. In the experiments, the effective elastic constants were determined with extensometers and tensometers and the stress distributions were found from the fringe pattern of a photoelastic test.

W. Schumann, Switzerland

**2943. Delchambre, R., Janssens, P., and Vanbeckbergen, Monique, Exact solution of the equation for loaded circular plates of linear variable thickness** (in French), *C. R. Acad. Sci. Paris* **250**, 26, 4271–4273, June 1960.

**2944. Hersch, J., A principle of the Thomson kind for the equilibrium of a clamped loaded plate** (in French), *C. R. Acad. Sci. Paris* **250**, 18, 2992-2994, May 1960.

**2945. Legendre, R., Stress distribution in a band or in a rectangle** (in French), *C. R. Acad. Sci. Paris* **250**, 19, 3108-3110, May 1960.

**2946. Bassali, W. A., Bending of a singularly loaded thin circular annulus with free boundaries,** *J. Mech. Phys. Solids* **8**, 2, 123-140, May 1960.

Series solutions are presented for the complex potentials and deflection of a thin circular annular plate of constant thickness under any system of concentrated forces or concentrated couples in equilibrium, the edges of the annulus being free. The limiting cases when the radius of the inner edge tends to zero or the radius of the outer edge tends to infinity are investigated and exact solutions in closed forms are deduced. Worked out in detail are the particular problems of circular ring plates subjected, respectively, to two bending couples, to two twisting couples, both applied at the ends of a diameter of a concentric circle, and to four forces applied at the ends of two perpendicular diameters of a concentric circle. The method of images is used to obtain the deflection of a sectorial plate bounded by two free arcs of concentric circles and two radii when the plate is acted upon by a concentrated load at an arbitrary point and is simply supported along the straight edges.

From author's summary by B. E. Gatewood, USA

**2947. Pister, K. S., and Dong, S. B., Elastic bending of layered plates,** *Proc. Amer. Soc. Civ. Engrs.* **85**, EM 4 (*J. Engng. Mech. Div.*), 1-10, Oct. 1959.

Large deflection of two or more thin bonded layers of different isotropic materials under transverse and in-plane forces is considered by the author. The theory, developed by neglecting the effect of transverse shear deformation, includes the effect of thermal gradient over the plate thickness and over the plate faces. The theory is an extension of Von Karman's theory, and two governing differential equations obtained contain terms which indicate the coupling effect introduced by the layered construction. The case when the layered plates have the same Poisson's ratio with different moduli of elasticity has been deduced. Expressions for transverse shear and resultant stresses are also obtained. D. N. Mitra, India

**2948. Kornishin, M. S., The flexure of sloping cylindrical panels and plates with pliant edges** (in Russian), *Izv. Kazansk. Fil. Akad. Nauk SSSR, Ser. Fiz.-Matem. i Tekhn. Nauk* no. 12, 91-100, 1958; *Ref. Zh. Mekh.* no. 9, 1959, Rev. 10520.

The flexure is investigated of slanting cylindrical isotropic panels with consideration for large deflections under the action of an evenly distributed load. The basic equations are solved approximately. Each equation goes through the required procedure laid down in the Bubnov-Galerkin method. The edges of the shell are assumed to be pliant and to have the capacity to move apart in a tangential plane. E. F. Burmistrov

*Courtesy Referativnyi Zhurnal, USSR*

**2949. Karapetyan, M. E., A single mixed problem on the bending of an elastic plate** (in Russian), *Trudl Tbilissk. In-ta Inzh. Zh.-d. Transp.* no. 31, 63-70, 1957; *Ref. Zh. Mekh.* no. 9, 1959, Rev. 10543.

An investigation is made, using N. I. Muskhelishvili's method, of the bending of an elastic thin homogeneous isotropic plate the middle surface of which occupies a finite singly-bounded region $T$ of a plane $xy$, delimited by a simple smooth closed curve $L = L_1 + L_2$, where part $L_1$ of the contour $L$ is rigidly clamped while the remaining part $L_2$ is free. For cases where region $T$ of the plate bounded by contour $L$ is reflected onto the interior of a unit circle by means of a polynome or a rational function, the problem of bending being considered merges with Gilbert's problem with discontinuous coefficients, the solution for which is known. There are misprints in the paper. M. P. Sheremet'ev

*Courtesy Referativnyi Zhurnal, USSR*

**Book—2950. Soare, M., Applications of finite difference equations for the calculations of shells [Aplicarea ecuatiilor cu diferente finite la calculul placilor curbe subtiri]**, Bucuresti, Editura Academiei Republicii Populare Romine, 1959, 401 pp.

In view of the large number of existing papers and considering the practical interest of thin shells and the complexity of calculation methods, this book has a synthetic character and uses a simple and readily accessible instrument of calculation.

Chapter I presents the general theory of thin shells. Chapter II introduces the finite difference method for the calculation of shells. Chapters III-VII deal with the membrane theory of shells of revolution, doubly curved shells, translational shells and shells with double directrix and directrix plane.

Chapters VIII and IX deal with the axisymmetrical bending theory of cylindrical and folded shell roofs.

Chapter X sketches the interpretation of model test results with the aid of finite differences.

In each chapter, the finite difference method is developed, seeking the best way of applying it to the particular type of shell.

Numerical examples and a large number of references complete this new book, which is of a great value for advanced students, research workers and designers. A. L. Petre, Roumania

**2951. Peterson, J. P., and Dow, M. B., Structural behavior of pressurized, ring-stiffened, thin-wall cylinders subjected to axial compression,** NASA TN D-506, 24 pp., Oct. 1960.

Axisymmetric mode of deformation of the cylinder mentioned in the title is analyzed based on the shell equation which is equivalent to that of a beam on elastic foundation subjected to thrust and lateral load, and the results are in good agreement with experiments. As is expected, for small axial load the deformation is restricted within the vicinity of the ring, but when the load approaches the critical value of buckling, the deformation waves are seen to extend over broader range of the cylinder, the damping ratio becoming nearer to unity. Y. Yoshimura, Japan

**2952. Rygol, J., The calculation of thin shells of revolution of variable thickness,** *Civ. Engng., Lond.* **55**, 649, pp. 1015, 1017, 1019, 1020, Aug. 1960.

Equilibrium equations for a thin shell of revolution loaded with forces and moments uniformly distributed along its edge are reduced to a fourth-order, ordinary differential equation. This is accomplished by neglecting shearing stress and tangent rotation terms and their first derivatives on the claim that these damp out quickly as one recedes from the edge. The ratio between two consecutive extreme moment values is shown to be only 0.043 for a spherical shell of constant thickness.

By assuming a simple expression in terms of two arbitrary parameters, which defines the shell thickness, the general solution of the differential equation is obtained. Thus four constants of integration plus two arbitrary parameters are involved. The first are evaluated from particular boundary conditions, the second by suitable selection to account for particular thickness variation.

Author concludes satisfactory solutions for shells of variable thickness can be obtained in two ways. Either replace the edge zone with a constant average thickness or suitably choose the parametric constants.

Shearing stress, normal stress, moment, and rotation relations are derived for edge force and edge moment loadings.

J. P. Vidosic, USA

**2953. Candela, F., General formulas for the calculation of stresses in paraboloid-hyperbolic shells** (in Spanish), *Ingenieria, Mexico* **30**, 2, 22–31, Apr. 1960.

Paper concerns same subject previously studied by author [AMR **8**(1955), Rev. 1945] but now presented with new developments. Hypar shells (as paraboloid-hyperbolic shells are called by author) are first considered in a general way and some peculiar cases are developed in succession. Several examples of reinforced-concrete hypar shells built in Mexico are given.

F. Correia de Araujo, Portugal

**2954. Pulos, J. G., and Buhl, J. E., Jr., Hydrostatic pressure tests of an unstiffened cylindrical shell of a glass-fiber-reinforced epoxy resin,** David W. Taylor Mod. Bas. Rep. 1413, 14 pp., Apr. 1960.

An unstiffened cylindrical shell of 32.67-in. mean diameter, built up of layers of an epoxy resin tape reinforced with glass fibers, was tested to collapse under external hydrostatic pressure. The shell structure failed at a pressure of 3735 psi, which is in good agreement with the pressure calculated from the elastic shell-buckling equations of von Mises. A longitudinal tear accompanied by some delamination of the layers was observed over the entire length of the cylinder.

From authors' summary

**2955. Mann, W., Analysis of north light shell roofs by the beam method with a consideration of transverse deformation** (in German), *Beton u. Stahlbeton.* **55**, 3, 64–68, Mar. 1960.

This article presents a two-step calculation of cylindrical north light shells; the first step is performed according to Lundgren's method assuming a rigid cross section. The second step involves a torsion calculation by setting the condition of the non-deformation of the window columns.

A detailed numerical example leads to results which agree well with the exact theory.

The presentation, very clear and concise, is intended for shell designers.

M. V. Soare, Roumania

**2956. Dulacska, E., Dome shells of complex surface over a polygonal base** (in Hungarian), *Mélyépítéstudományi Szemle* **10**, 1, 38–40, Jan. 1960.

No lateral reactive forces act on the boundary of shell. The shell consists of parts having different curvatures, and each part is an elliptical paraboloid. The neighboring parts are tangentially joined to one another. For this reason, the sectional forces (membrane forces) can be expressed in a closed form; nevertheless, bending moments arise along the joining lines. By means of an approximate computation, author demonstrates that, because of these bending moments, no extra reinforcement is necessary. A numerical example is given for the case when the base is an equilateral triangle.

J. Barta, Hungary

**2957. Mihailescu, M., and Mihailescu, Y., Calculation of translational shells by the relaxation method** (in French), *Rev. Mécan. Appl.* **5**, 5, 669–694, 1960.

The relaxation method is applied for studying thin translational shells in the membrane theory. The directrices are arcs of circle and the covered surface is a rectangle whose ratio of sides is 2, 1.3 and 1.

Shells supported on shear diaphragms, shells without opening at the vertex or with skylight and shells supported at corners are examined. The results are tabulated in order to be directly used by designers.

M. Soare, Roumania

**2958. Miller, P. R., Free vibrations of a stiffened cylindrical shell,** Aero. Res. Counc. Lond. Rep. Mem. 3154, 38 pp., 1960.

Lagrange equations and characteristic equations for natural frequencies are obtained for uniform cylindrical shell and for a unifomly stiffened shell (where frame and longeron spacing is small compared with nodal spacing) *in vacuo*. Shear flexibility is neglected but rotatory inertia is included in the case of the stiffened shell. Exposition is unusually lucid and there is an extensive survey and critique of work in this field up to 1955.

S. H. Crandall, USA

**2959. Ionov, V. N., Stress-strain conditions of a shell with zero curvature** (in Russian), *Inzhener. Sbornik, Akad. Nauk SSSR* **29**, 63–76, 1960.

Author proposes a method of analysis of cylindrical and conical shells under arbitrary loads. The method, based on the variational principle of minimum strain energy, is free of Kirchhoff-Love assumptions. The stress tensor is written as a sum of a principal tensor and a correcting tensor. Stress-strain relations involve two elastic constants, temperature, and a function of plasticity.

A numerical example is solved with the aid of an electronic computer.

R. Schmidt, USA

**Book—2960. Aleksandrov, A. Ya., Bryukker, L. E., Kurshin, L. M., and Pruskakov, A. P., Analysis of three-layered panels [Raschet trekhsloinykh panelei],** Moskva, Gosudarstvennoe Nauchno-tekhnicheskoe Isdatel'stvo Oborongiz, 1960, 269 pp. 15 r 20 k.

This is essentially a handbook containing almost all known information pertaining to sandwich construction. Topics treated include the stability of (a) 3-layered plates with light isotropic cores, (b) 3-layered plates with light orthotropic cores, (c) 3-layered plates with orthotropic cores and orthotropic facings, and (d) curved 3-layered panels with light isotropic cores. Equilibrium considerations with treatments of stresses and deflections are given for these same structures when subjected to either uniform or linearly varying lateral loads. The last portion of the book deals with these same structures when subject to combined transverse and in-plane loadings.

In general, only results are presented with a minimum of derivations. Results are conveniently given in the form of graphs and tables. There is little original work presented but the book does give a unified treatment of the entire sandwich construction problem. It is well referenced.

W. Nash, USA

**2961. Chang, C. C., and Fang, B. T., Initially warped sandwich panel under combined loadings,** *J. Aerospace Sci.* **27**, 10, 779–787, Oct. 1960.

Based on small-deflection theory, differential equations for the elastic bending of an orthotropic weak-core sandwich panel with small initial warping are derived by the variational energy method. The applied loads consist of arbitrarily distributed transverse loads and eccentrically applied edge loads and/or edge moments. For the case of a simply supported rectangular panel, solutions of the differential equations are obtained in the form of double Fourier series. As a practical example, numerical values of the maximum stresses and deflections for a square sandwich panel are calculated and presented in the form of graphs.

A simple design criterion is suggested for considering approximately the effect of initial warping and transverse pressure. The method is particularly useful for preliminary design.

From authors' summary by J. W. Clark, USA

**2962. Akao, S., Analysis of shear lag problems by means of difference equations,** *Technol. Rep. Osaka Univ.* **9**, 91–99, Mar. 1959.

Stress analysis in the plate-stiffened $(n+1)$ ribs is made by solving simultaneous differential equations involving $(n-1)$ statically indeterminate elements. This is simplified by using groups of orthogonal statically indeterminate force functions. Finite difference equations which give these eigenfunction groups are obtained. Author gives tables of eigenfunction groups for the plate when $n=3$ to $n=7$. The effects of the lateral ribs and Poisson effects are neglected.

D. N. Mitra, India

**2963. Mazurkiewicz, Z., The problem of deflection surface of rectangular isotropic and non-homogeneous plate** (in English), *Bull. Acad. Polonaise Sci.*, (IV) **8**, 1, 5–13, 1960.

Author presents a general method for obtaining the deformed surface of isotropic plates with variable flexural rigidity and arbitrary load acting perpendicularly to the middle-surface, assuming the plate rectangularly shaped and simply supported. Stating the differential equation for the elastic surface, author gives an implicit solution by a nonhomogeneous Fredholm integral equation, applying the Green function. For practical application this function is represented by a double Fourier series, obtaining thus a system of nonhomogeneous linear equations for the unknown coefficients. In some particular cases closed solutions are possible.

H. Beer, Austria

**2964. Szelagowski, F., An orthotropic plate supported on two opposite boundaries with uniformly loaded rectangular field parallel to the plate boundaries,** *Bull. Acad. Polonaise Sci.* **8**, 4, 167–177, 1960.

The method of Fourier series is applied to finding the static deflection, the bending and the torsion moments in a rectangular orthotropic plate supported on two opposite edges and carrying a uniform load on its rectangular part.

The comparatively complicated resulting formulas are of significance for technical purposes, e.g. at testing several beam-grid constructions.

V. Vodicka, Czechoslovakia

## Buckling

(See also Revs. 2910, 2951, 2960)

**2965. Barta, J., Beck's stability problem and related problems** (in German), *Acta Techn. Acad. Sci. Hungaricae, Budapest* **31**, 1/2, 241–259, 1960.

M. Beck's stability problem [AMR **6**(1953), Rev. 80] consists of determining the critical value of compressive loading force which acts tangentially at the free end of thin elastic bar, the other end of which is fixed. This is a case of nonconservative load. Principles for treatment of such stability problems are due to H. Ziegler [see feature article by H. L. Langhaar: AMR **11**(1958), p. 585]. In present paper, Beck's process is recapitulated and illustrative examples are given for cases where the loading force is nonconservative, or where the point of application of loading force is not connected to a certain point of structure. An interesting case is shown where the equilibrium under action of a tensile loading force becomes unstable.

S. Sarkadi Szabo, Hungary

**2966. Webber, J. P. H., and Houghton, D. S., Thermal buckling of a free circular plate,** Coll. Aero., Cranfield, Note 105, 7 pp. + charts, Aug. 1960.

The buckling of a free circular plate subjected to a rotationally symmetrical temperature distribution is investigated experimentally and by means of an approximate small-deflection energy analysis. There is favorable agreement.

E. H. Mansfield, England

**2967. Darevskii, V. M., and Kukudzhanov, S. N., Torsional stability of an orthotropic shell under internal pressure,** *Soviet Phys.-Doklady* **3**, 6, 1271–1274, June 1959. (Translation of *Dokladí Akad. Nauk SSSR (N.S.)* **123**, 1, 49–52, Nov. 1958 by Amer. Inst. Phys., Inc., New York, N. Y.)

Authors investigate stability in torsion of internally pressurized orthotropic cylinders using an equation similar to Donnell's equation for isotropic shells. By means of various approximations the following results are obtained. For pure torsion or external lateral pressure the critical stresses are given by formulas similar to those for moderate length isotropic cylinders, namely

$$T_{cr}/2\pi R^2 t = 0.74\left(\frac{E_2}{1-\nu_1\nu_2}\right)^{5/8} E_1^{3/8} (t/R)^{5/4} (R/L)^{1/2}$$

$$q_{cr} R/t = \frac{2\pi}{3/6}\left(\frac{E_2}{1-\nu_1\nu_2}\right)^{3/4} E_1^{1/4} (t/R)^{3/2} (R/L)$$

where the subscripts 1 and 2 refer to the axial and circumferential directions, respectively. For sufficiently high internal pressure, the approximate interaction curve is given by

$$(T/T_{cr})^2 = q/q_{cr}$$

a result which is the same as for an isotropic cylinder.

P. Seide, USA

**2968. Pope, G. G., The initial instability of an elastic sheet reinforced by stringers and skew ribs,** *J. Roy. Aero. Soc.* **64**, 596, 489–491 (Tech. Notes), Aug. 1960.

## Vibrations of Solids

(See also Revs. 2831, 2844, 2852, 2854, 2958, 2984, 3036, 3060, 3295, 3360, 3361)

**Book—2969. Bishop, R. E. D., and Johnson, D. C., The mechanics of vibration,** New York, Cambridge University Press, 1960, xii + 592 pp. $19.50.

This book differs greatly from the usual textbooks on vibrations. Right at the beginning the idea of receptance is introduced, the first chapter (called Introduction) leads to the tables known from an earlier publication of the authors ["Vibration analysis tables," Cambridge Univ. Press, 1956]. Chapter II deals with the generalized coordinates of Lagrange, chapter III with systems of a finite degree of freedom, where both concepts of the preceding chapters are used. Next, the taut string (IV), the shaft (VI) and the beam (VII) follow; in the latter chapter the tables of the earlier work which can be used for the calculation of a beam of several sections are included. Chapter X discusses free vibrations with (light) damping. The remaining chapters deal with more or less "real" systems: V with Rayleigh's principle, Dunkerley's formula, etc.; VIII and IX with viscous and hysteretic damping, XI with transient vibrations (linear theories, like the rest of the book).

In the preface a kind of an excuse is given for the matrix notation not being used; in the reviewer's opinion this is a fault in a work of this standard. On the other hand the topics dealt with are so clearly presented that the book may be recommended to both physicists and engineers.

K. Marguerre, Germany

**2970. Gershuni, G. Z., and Zhukhovitskii, E. M., Forced vibrations in an elasto-plastic system** (in Russian), *Fiz. Metallov i Metallovedenie* **6**, 2, 339–346, 1958; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5578.

The forced vibrations beyond the limit of elasticity in an elasto-plastic system, with hysteresis and friction in evidence, are investigated. It is assumed that the characteristic of the elasto-

plastic force has the form of a parallelogram passing through a basis 0, with one pair of the sides having an angular coefficient $k_1$ (the modulus of elasticity), while the other pair a coefficient $k_2$ (the modulus of hardening); their relation being $\alpha = k_2/k_1$. The equation for the forced vibrations of a system with linear resistance and harmonic perturbation is reduced to dimensionless variables and takes the form of

$$\ddot{x} + \beta\dot{x} + f(x) = g \sin (pt + \varphi) \quad [1]$$

A periodic solution is sought for this equation with a period of $2\pi/p$. The problem is solved by means of the Bubnov-Galerkin method, when the solution approximates the function

$$\overline{x}(t) = (\delta/2) - a \cos pt \quad [2]$$

where

$$a = 1 + \frac{1 + \alpha}{1 - \alpha}\frac{\delta}{2}, \qquad \delta = (\Delta/x_m) \quad [3]$$

Here $x_m$ is the limit of elasticity, attained through time $\tau$ which is adopted for the unit of displacement, while $a$ is the residual displacement. Three equations are derived for the determination of three magnitudes $a$, $\tau$, $\varphi$ as functions of the amplitude and frequency of the perturbing force and the parameters of the system. The solutions are carried out first of all for the case where there is no linear resistance ($\beta = 0$) and then for the case where there is, but for a single special case only ($\alpha = 0$, $\beta \neq 0$); graphs for the amplitudes are drawn. The course taken by these curves indicates the considerable absorption of energy evoked by plastic hysteresis; the asymmetry of the curves increases with the decrease of $\alpha$. With increase of friction there is an increase in the resonance frequency and the plastic effects are lifted in part. A boundary case is investigated in which the characteristic of the expanding force has the form of a rectangle. For this case not only an approximate but also a precise solution is obtained by use of the method of "cementing together." In carrying this out it was shown that the smaller the friction the closer to each other are the two solutions; with no friction the two solutions coincide.

A. N. Obmorshev
*Courtesy Referativnyi Zhurnal, USSR*

**2971. Keropyan, K. K., Determination of the frequencies of the free vibrations of elastic systems** (in Russian), *Trudi Rostovsk.-N/D. Inzh.-Stroit. In-ta* no. 6, 159–179, 1958; *Ref. Zh. Mekh.* no. 9, 1959, Rev. 10571.

A method is described for the determination of several first frequencies of the free vibrations of elastic systems without having to solve the differential equations. To attain this objective a further development is undertaken of the method of the spectral function [see S. A. Bernshtein, "A new method for the determination of the frequencies of the vibrations of elastic systems," Moscow, Izd-vo Voen.-Inzh. Akad. RKKA im. V. V. Kuibysheva, 1939]. Proof is furnished for a number of theorems which offer the possibility of evaluating the approximation of the roots of the transformed polynomes to the roots of the spectral function. Four inequalities are obtained enabling determinations to be made of the first four frequencies for the vibrations with a two-sided evaluation (from the top and from the bottom). Tables are furnished for the values of the coefficients of the formulas which are necessary for the calculations. A numerical example is given for the determination of the frequencies of the vibrations of a mono-cantilever beam, loaded with three masses; the second and the third frequencies obtained by the author's method differ from the exact values by 2.32% and 0.68% respectively.

A. P. Yakovlev
*Courtesy Referativnyi Zhurnal, USSR*

**2972. McKinney, W. T., Matrix equations for the determination of beam natural frequencies,** *J. Soc. Indust. Appl. Math.* **8**, 3, 436–457, Sept. 1960.

Author treats coupled flexural, shear and torsional vibrations, based on the usual elastic axis concepts. (Rotatory and torsional inertia effects are included.) Paper delineates information required for proceeding from beam section data to characteristic matrix. Methods of imposing boundary conditions are illustrated for cantilever, free-free, fixed-fixed, and hinged-hinged beams. Equations for calculation of influence coefficients are summarized, and convenient data preparation and calculation procedures are outlined.

L. Dyer, USA

**2973. Chuvikovskii, V. S., Flexo-torsional vibrations of nonprismatic bars, taking into account the shear deformation caused by transverse shear forces and energy dissipation** (in Russian), *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk* no. 3, 72–77, May/June 1959.

General vibrations of a variable section beam (ship hull), taking into account internal and external linear damping, are investigated. The method of finite differences is applied to the generalized case of lumped masses replacing the beam. The inertia effect of the fluid is replaced by the appropriate adjoined mass for each segment. A consecutive recurrence method, similar to Myklestad's method, is used to determine the forces acting on each lump.

The method seems to be particularly useful for electronic computers.

W. Fiszdon, Poland

**2974. Oseled'ko, A. I., Calculations for the frequencies of the natural vibrations of a rectilinear beam lying on an elastic base and compressed by axial forces** (in Russian), *Sb. Tr. Voronezhsk. Inzh.-Stroit. In-ta* no. 4, 127–131, 1958; *Ref. Zh. Mekh.* no. 9, 1959, Rev. 10585.

Author analyzes the formula for the frequencies of the natural vibrations of a beam on an elastic base, compressed by longitudinal forces. The number of semiwaves along the length of the beam is found at which the frequency will reach its minimum; both the coefficient of the elastic base and the compressive force are assigned to make this possible. In the absence of an elastic base when the compressive force is greater than the critical, a corresponding form of vibrations does not exist.

V. E. Breslavskii
*Courtesy Referativnyi Zhurnal, USSR*

**2975. Rabinovich, B. I., On the equations of elastic vibrations of thin-walled shafts partially filled with liquid** (in Russian), *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk* no. 4, 63–68, July/Aug. 1959.

The equations describing small free vibrations of a thin-walled hollow tube partly filled with liquid are formulated for the particular case where the external body force acts along the axis of the tube. The motion of the liquid is potential, and the usual boundary conditions for gravity waves of small amplitude are imposed at the free surface. On the wetted internal wall of the tube, the normal velocity of the liquid and the wall are the same, so that the boundary conditions for the potential flow problem involve the unknown velocity of the tube. Making use of Green's functions, the author expresses the potential function in terms of the boundary velocities. He then finds the pressure exerted on the tube by the liquid, and hence is able to substitute the resultant pressure forces and moments into the equation of flexural and torsional vibrations of the tube, obtaining an integrodifferential equation for the free vibrations of the tube. No solutions are given.

E. Saibel, USA

**2976. Tondl, A., Influence of elastically supported frames on the stability of rotating shafts with consideration of the internal**

**and external damping** (in German), *Öst. Ing.-Arch.* **14**, 2, 93-99, June 1960.

Author considers an idealized system, consisting of a disk on a weightless shaft, connected to an elastically supported rigid frame, the whole system being fully symmetrical, so that all motion takes place in a plane perpendicular to the shaft. He assumes furthermore that the flexibility of the shaft and the elastic supports is linear and equal in all directions. Damping is introduced by two linear functions, one for the relative motion between shaft and frame and the other for the motion of the frame. This problem results in a system of four ordinary differential equations of second order with constant coefficients. This system is solved by successive iterations and author shows that the limit of stable motion is higher than the eigen frequency of the system, increase being partly due to internal and partly due to external damping.

F. I. Niordson, Denmark

**2977. Bolotin, V. V., Investigation of the vibrations of rollers with different principal stiffnesses during deflection** (in Russian), Calculations for strength, no. 2, Moscow, Mashgiz, 1958, 302-312; *Ref. Zh. Mekh.* no. 9, 1959, Rev. 10588.

A system of two differential equations is put forward to describe the vibrations of a rotating roller with different values for the principal stiffnesses during deflection. The problem undergoes a preliminary analysis in a linear setting; the conditions for stability are investigated and also the influence of friction on the stability. In the nonlinear examination account is taken of the nonlinearity produced by the action of the longitudinal force, combined with the movability of one of the ends of the roller, the nonlinear inertia of the mass and also the nonlinear damping. Special solutions of the nonlinear system and their stability are examined.

F. M. Dimentberg
*Courtesy Referativnyi Zhurnal, USSR*

**2978. Bohm, F., Torsional vibrations in gears** (in German), *Öst. Ing.-Arch.* **13**, 2, 82-103, 1959.

Gears in mesh form a vibrating system in which the inertia is in the wheels and the stiffness in the teeth. This stiffness, in spur gears, depends upon the number of teeth in contact and on tooth position. The stiffness variation can cause self-excited vibration to develop in certain speed ranges if damping is small enough. Vibration of the variable stiffness system can also be excited by transmission errors; the amplitude of these vibrations is found by an interation process. A numerical example is given. Finally, the effects of backlash are examined by a phase-plane method.

Reviewer believes this is the most complete theoretical treatment of the gear oscillation problem yet published.

D. C. Johnson, England

**2979. Waldersee, J., Response of a seismically mounted machine tool to a transient site vibration,** *Austral. J. Appl. Sci.* **11**, 2, 250-260, June 1960.

Title term "site vibration" means vibration of support or foundation on which machine is suspended. System analyzed has same elements and arrangements as spring-mass vibration absorbs. The support excitation is damped harmonic motion, $x_s = A \exp(-\Delta t) \sin \omega t$. Laplace transformation method is used to analyze motion of seismic mass (the machine) and to analyze motion of cutting element. Special cases for resonance between seismic suspension and support as well as between cutting element and machine frame are considered. Results obtained agree closely with analog computer solutions.

W. J. Worley, USA

**2980. Mead, D. J., Criteria for comparing the effectiveness of damping treatments,** Univ. Southampton, Dept. Aero. & Astron. Rep. 125, 31 pp. + figs., June 1960.

In this paper, expressions are derived for the response of simple vibrating systems, from which criteria have been deduced to indicate the effectiveness of a damping treatment in attenuating the response. The criteria include factors by which the treatment increases the mass and stiffness of the system, together with the loss factor increment. The response quantities considered include bending stresses, accelerations, inertia forces and sound transmission associated with simple vibrating plates under harmonic and random excitation. Coincidence sound transmission is also briefly considered. It is shown that whereas the mass and loss factor increase is always advantageous, a stiffness increase in some instances is detrimental.

As an example, three different commercial treatments are compared on the basis of some of the criteria. With low treatment weights, the treatment providing the highest loss factor is superior judged by each criterion, but at higher weights according to some criteria a treatment having a lower stiffness, density and loss factor is more effective. The existence of optimum treatment weights for maximum effect upon the response is also shown by some criteria.

From author's summary

**2981. Lancaster, P., Free vibrations of lightly damped systems by perturbation methods,** *Quart. J. Mech. Appl. Math.* **13**, 2, 138-155, May 1960.

The problem of small vibrations of a conservative mechanical system is formulated. A method is developed by means of perturbation theory for the estimation of frequencies, rates of decay, and modes of vibration of the system when small damping forces are introduced. The analysis is based on a prior knowledge of the natural frequencies and principal modes of the undamped system. Special attention is given to the cases in which some natural frequencies of the undamped system are exactly equal, and those in which oscillations occur about a position of equilibrium which is critically stable. When several of the natural frequencies are nearly equal, a method of solution is given which can be applied when the variation in these frequencies is related to the magnitude of the damping forces in a particular way.

From author's summary by E. J. McBride, USA

**2982. Niblett, D. H., and Wilks, J., Dislocation damping in metals,** *Advances in Phys.* **9**, 33, 89 pp., Jan. 1960.

The mechanical energy of a vibrating solid is converted quite rapidly into heat, even if the body is completely isolated from its surroundings. This energy transfer, together with the consequent damping of the vibrations, is said to arise because of the presence of *internal friction.* Measurements of internal friction provide valuable information about the structure of the solid state and a wide variety of work has been reported in the last twenty years. Several sources of internal friction in metals have been well understood for some time (Zener 1948), but it is only recently that much information has become available on what is often the principal source of damping in metals, namely the motion of dislocations. Authors review the various experimental results and consider to what extent they can be explained by current theories.

From authors' summary

**2983. Klotter, K., and Kreyszig, E., Limiting amplitudes of self-excited vibrations for small energy exchanges** (in German), *ZAMM* **40**, 1/3, 55-61, Jan./Mar. 1960.

The method of slowly varying amplitudes is applied to a second-order differential equation of vibrations with a small nonlinear damping term which is a product of a polynomial expression of the displacement and the square of the velocity. The second-order differential equation transforms into a system of first-order equations for the amplitude $a$ and the phase angle $b$. Replacing the amplitude and phase angle rates by their average over a full cycle furnishes a first-order approximation. An explicit solution for the

amplitude $a_G$ of the limit cycle is presented for the special case that the displacement-dependent factor in the damping term can be represented by the difference of a constant and an even power of the displacement. The stability of $a_G$ is proved and the phase angle is found independent of the amount of energy exchange. An improved solution is derived by replacing the averages of the amplitude and phase angle rates by Fourier series (Kryloff-Bogolyubov method). Its solution gives the periodic deviation of the amplitude from its average while the phase angle is still assumed constant. G. W. Braun, USA

## Wave Motion and Impact in Solids

(See also Revs. 2939, 3033, 3041, 3295, 3317)

**Book—2984. Bland, D. R., Vibrating strings—An introduction to the wave equation**, London, Routledge and Kegan Paul, 1960, v + 95 pp., 5s. (Paperbound)

Author presents a discussion of the wave equation in terms of the transverse vibrations of finite and infinite strings. Clear concise treatment of normal modes, solution by Fourier series, D'Alembert's solution, reflection of waves at discontinuities and forced vibrations in presence of air resistance is given. Many illustrative examples and problems, with answers, are provided. Book is suitable for self-study and should be useful to undergraduate engineers. T. P. Mitchell, USA

**2985. Cassity, C. R., Stress waves in solids**, *J. Appl. Phys.* **31**, 8, 1377–1381, Aug. 1960.

Author describes a number of experiments in which colliding transient stress disturbances generated by exploding charges produce tensile fractures in metallic bodies having various geometric configurations. He concludes that the fractures arise from tensile components developed within the original waves rather than from tensile waves created by reflections at boundaries, a view more commonly held. In reviewer's opinion the data are not definitive. J. Rinehart, USA

**2986. Filippov, A. F., A three-dimensional problem of diffraction of an elastic wave at a sharp edge**, *Appl. Math. Mech.* (*Prikl. Mat. Mekh.*) **23**, 4, 989-996, 1959. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

Using the procedures presented by P. Frank and R. von Mises, in "Differential and integral equations in mathematical physics," part 2, ONTI, 1937, for the defraction of acoustic waves in a fluid, the author develops an exact solution for the defraction of a transient plane elastic wave propagated without resistance in three-dimensional space and striking against a rigid edge in the form of a half-plane. The method of functional-invariant solutions is used. F. E. Reed, USA

**2987. Kosachevskii, L. Ia., On the propagation of elastic waves in two-phase media**, *Appl. Math. Mech.* (*Prikl. Mat. Mekh.*) **23**, 6, 1593–1604, 1959. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

Paper is concerned with the study of the simplest case of motion, which is the propagation of elastic waves in a homogeneous isotropic medium consisting of a solid and a fluid phase. The problems of the reflection of plane waves and surface waves at the free boundary of the half-space are solved. It is shown that the stress-strain relations established by Frenkel are equivalent to the analogous relations proposed by Biot and that the equations of motion of the latter are more general.

From author's summary by F. Krupka, Czechoslovakia

**2988. Grosyuk, D. S., Scattering of sound waves by the uneven surface of an elastic body**, *Soviet Phys.-Acoustics* **6**, 1, 26–29, July/Sept. 1960. (Translation of *Akust. Zh., USSR* **6**, 1, 30–33, Jan./Mar. 1960 by Amer. Inst. Phys., New York, N. Y.)

Paper considers a longitudinal wave (wave vector $k$) in a fluid incident upon a solid boundary represented by $z = a\cos gx$, where $ka \ll 1$. The total scattered spectra in the reflecting and refracting media are expressed as the infinite sums of plane waves. The amplitudes of these waves for the zero-order (plane interface) and first-order approximations are given in algebraic form.

Numerical values of the first-order amplitudes are plotted and confirm that critical angles and waves at the interface occur analogously to the case of the plane interface.

There is a misprint of − for + in the expression for $\chi_n$ on p. 27. Reviewer believes paper would have been enhanced if the zero-order amplitudes for the same fluid-solid interfaces had also been presented so that a clear impression of relative magnitudes could quickly be gained. M. J. P. Musgrave, England

**2989. Cristescu, N., On the propagation of elasto-plastic waves for combined stresses**, *Appl. Math. Mech.* (*Prikl. Mat. Mekh.*) **23**, 6, 1605–1612, 1959. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

Paper studies the problem stated in the title on the basis of equations established by Rakhmatulin. For example, it is shown that the combined stress propagates in a body by two groups of ordinary waves, if the body remains elastic, i.e. for a shock which does not exceed the elastic limit. For transition of the elastic limit, plastic strain propagates in the body by two types of waves of combined stress which propagate in the body faster than the usual plastic waves. The study is rather qualitative, since the theory of small elasto-plastic deformations has not yet been verified experimentally for dynamic problems.

F. Krupka, Czechoslovakia

**2990. Agamirov, V. L., and Vol'mir, A. S., Behavior of cylindrical shells under dynamic loading of all-sided pressure or axial compression** (in Russian), *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk* no. 3, 78–83, May/June 1959.

Behavior of a circular cylindrical shell under the effect of dynamic pressure from all sides is investigated, taking nonlinearity into account.

Initial deflection of shell is considered and only inertia loads due to radial displacements are included. A three-parameter deflection function is assumed and the Bubnov-Galerkin method of solution is used. Deflection time curves are given for a particular cylinder for four loading velocities, showing the critical pressures and the dynamic behavior of the shell. It is reported that the experimental results confirmed the theoretical calculations.

The case of axial compression is treated similarly.

W. Fiszdon, Poland

## Soil Mechanics: Fundamental

(See also Revs. 2910, 3325, 3349)

**2991. Fellner, L., Critical condition for sliding** (in German), *Bauingenieur* **34**, 9, 353–357, Sept. 1959.

Based on Fröhlich's and author's former works, the simplest formula to obtain the safety factor for the failure of inclined surface is derived. The safety factor is calculated assuming a circular sliding surface.

Necessary figures and tables are given. New method is explained by numerical examples. T. Mogami, Japan

**2992. Schumann, W., Analysis of upper layer thickness from dynamical tests of soil with depth-dependent shear modulus** (in German), *Bauingenieur* **35**, 5, 167–170, May 1960.

Author considers an elastic layer of soil resting on a firmer foundation. The elastic moduli of the layer increase linearly with depth. The average velocity of waves propagating from a source to a receiver on the surface of the layer is investigated. Author shows that the thickness of the layer and its elastic properties may be determined using the relation between the average wave velocity and the distance from the source to the receiver.

M. P. Bieniek, USA

**2993. Biarez, J., and Biguenet, G., Mechanical properties of a soil after injection of a solution of silicate and sodium bicarbonate** (in French), *C. R. Acad. Sci. Paris* **250**, 26, 4286–4288, June 1960.

**2994, Heurtaux, J., On certain singular solutions of the equations of limit equilibrium of non-cohesive soils** (in French), *C. R. Acad. Sci. Paris* **249**, 23, 2489–2491, Dec. 1959.

**2995. Biarez, J., and Scheuch, G., Scale model for measuring the foundation stresses under a foundation on non-saturated clay** (in French), *C. R. Acad. Sci. Paris* **249**, 25, 2719–2721, Dec. 1959.

**2996. van Thach, N., Approximate calculation of the repartition of stresses on a retaining screen in a cohesive medium with small internal friction** (in French), *C. R. Acad. Sci. Paris* **251**, 3, 332–334, July 1960.

**2997. Shestakov, V. M., Filtrating stability of sand slopes** (in Russian), *Gidrotekh. Stroit.* **29**, 10, 41–44, Oct. 1959.

Laboratory investigations and field observations on existing earth embankments have established a new method for determining the shape of the slope after its deformation under hydrodynamic action. Appropriate relations and charts are given.

Based on experimental data, in the case of slopes protected by graded filters, following condition for avoiding seepage is set down:

$$D_{10}/d_{80} \quad 15....20,$$

($D$ for graded filter; $d$ for protected material).

For greater $D_{10}/d_{80}$ ratios, critical discharge values have been observed beyond which hydrodynamic effect starts.

A method for establishing the required thickness of graded filter is also presented.

R.-J. L. Bally, Roumania

**2998. Osterman, J., Views on the stability of clay slopes** (in English), *Geologiska Foreningens i Stockholm Forhandingar* **82**, 3, 346–366, 1960.

Some aspects are presented on the estimation of the stability of clay slopes. Among other things, the structural changes of soil elements, exposed to shear, and their influence on strength are discussed. The relation between the soil plasticity and the ratio between strength and over-burden pressure is touched upon, and the influence of water pressure and seepage on the latter.

Further, the difference between the condition of equilibrium and that of stability is discussed.

From author's summary

## Soil Mechanics: Applied

(See Revs. 2996, 3062, 3330)

## Processing of Metals and Other Materials

(See also Revs. 2919, 3259)

**2999. Garudachar, B. N., and Petersen, H. A., Roll-force and torque coefficients for hot-strip steel mill,** *ASME Trans.* **82** D (*J. Basic Engng.*), 3, 683–688, Sept. 1960.

This paper provides results of an analytical and computer investigation to determine the numerical coefficients involved in the roll-force and roll-torque equations pertaining to a single stand in a multistand, tandem, steel rolling mill. The coefficients for a typical hot mill are compared with those obtained for a typical cold mill. The principles of gage control are discussed. The theories on flat-strip rolling are reviewed briefly.

From authors' summary by J. Frisch, USA

**3000. Ford, H., and Alexander, J. M., Rolling hard materials in thin gauges; basic considerations,** *J. Inst. Metals* **88**, 193–199, 1959–60.

Paper concerns two problems, i.e. limiting reduction (problem 1) and limiting thickness (problem 2) in cold rolling of thin, hard strip. Previous works of Stone, Hill and Longman, and Sims on problem 1 are reviewed and adequacy of the basic concepts is examined from the practical viewpoint. It is concluded that considerable extension of the fundamentals is required before a fully reliable calculation method can be realized.

As regards problem 2, a new theory is proposed for predicting the minimum thickness that it is possible to roll. The theory interprets problem 2 as that of incipient plastic deformation at one point in the strip, and leads to a minimum thickness equation based on elastic conditions in the strip, in contrast to Stone's equation. Later author hopes to present experimental work to to check accuracy of predictions of present theory.

Y. Yamada, Japan

**3001. de Kazinczy, F., and Backofen, W. A., Influence of hot-rolling conditions on brittle fracture in steel plate,** Ship Structure Committee Rep. 126, 26 pp., Nov. 1960.

Steel plates processed according to a conventional and controlled (low finishing temperature) rolling practice were studied to establish reasons for a superior notch toughness in the controlled-rolled product. The lower transition temperature (Charpy V-notch 15 ft-lb) in plates investigated is derived largely from a smaller ferrite grain size. Experimental evidence was also obtained to indicate that a part of the improvement results from a microfissuring in the plane of the plate at the notch root, with the effect that stress triaxiality is relieved and transition temperature depressed. The origin of the flaws responsible for the fissures was not determined with certainty, but there was good indication that they were simply inclusions in a fiber structure too fine in scale for observation with normal metallographic techniques. The criteria presented for microfissuring were: (1) a flaw structure dispersed on a scale no greater than the size of the plastic volume from which the brittle crack originated; (2) a ratio of the critical fracture stress in thickness ($Z$) to rolling ($R$) direction no greater than about 1/2. The necessary fracturing anisotropy was favored by more intense fibering (lower $\sigma_Z$) and finer grain size (higher $\sigma_R$). The criteria were most nearly satisfied by controlled-rolled plate. Correlations between $\sigma_Z$ and visible inclusion content showed the decrease in $\sigma_Z$ to be paralleled by more elongated inclusions, as if visible changes are indicative of changes in the fine-scale fiber structure.

From authors' summary

**3002. Herenguel, J., Lelong, P., and Moisan, J., Influence of heating before rolling (soaking) on the change of the structure and the properties of semi-continuously hardened sheets of industrial**

**aluminium** (in French), *C. R. Acad. Sci. Paris* **250**, 12, 2200-2202, Mar. 1960.

**3003. Sokolovsky, W., Longitudinal flow of plastic matter between two non-circular cylinders** (in French), *C. R. Acad. Sci. Paris* **249**, 25, 2713-2715, Dec. 1959.

**3004. Wheeler, W. H., Indentation of metals by cavitation,** *ASME Trans.* **82** D (*J. Basic Engng.*), 1, 184-194, Mar. 1960.

The first stage in cavitation damage—indentation where cavitation bubbles collapse near the metal surface—was observed and measured by Knapp. The author suggested that the high temperature of indentation points, caused by the absorption of work in their formation, should create conditions favorable to energetic local chemical action between water and metal. The brittle oxide films thus produced would be ruptured by later blows, resulting in erosion. Photomicrographs taken in monochrome and color show the form and texture of indentations with associated corrosive oxidation. Parallel experiments were made with toluene whose inertness permitted the elimination of chemical factors. The special features of the microscope technique employed to get the photographs are explained. From author's summary

**3005. Kuznetsov, D. P., Calculations for the forces exerted during the cold pressing of hollow cylindrical components** (in Russian), New features in the forging-punching works in Leningrad, Leningrad, 1958, 137-153; *Ref. Zh. Mekh.* no. 9, 1959, Rev. 10630.

Stresses and forces making their appearance during the process of cold three-dimensional pressing by means of extrusion are investigated. The problem is solved by the methods of lines of slide. A study is made of the toughening of the metal in the course of pressing, based on experimental curves. Formulas are given for the computation of the stresses and forces exerted in the pressing. The calculation data are compared with results of experiments made with prepared specimens in aluminum, brass and steel of low carbon content. A. D. Tomlenov

*Courtesy Referativnyi Zhurnal, USSR*

**3006. Averkiev, Yu. A., The procedure used for taking into account the toughening and analysis of operations involving change of form in cold pressing** (in Russian), Machines and technology of the treatment of metals by pressure (MVTU 79), Moscow, Mashgiz, 1957, 91-98; *Ref. Zh. Mekh.* no. 9, 1959, Rev. 10633.

Taking as an example the pressing of hollow cylindrical blanks the author studies the influence of the toughening of the material on the magnitude of the stresses when treating metals by means of pressure. Both the power and linear principles of toughening are used. In addition, the influence of the toughening is considered by the replacement of the limit of yield in the formulas for the stresses without toughening by a magnitude greater than the limit of yield. An evaluation is given for the discrepancies in the results obtained with different methods for taking the toughening into account. V. V. Moskvitin

*Courtesy Referativnyi Zhurnal, USSR*

**3007. Makarevich, A. I., The mean pressure of the yield of metal in the recess of a press of an annular type** (in Russian), *Sb. Nauchn. Tr. Fiz.-Tekhn. In-ta Akad. Nauk BSSR* no. 4, 112-123, 1958; *Ref. Zh. Mekh.* no. 9, 1959, Rev. 10634.

An investigation is made of the plastic stressed state which appears when metal is pressed in an annular press. It is assume that the direction of the principal normal stresses coincides with the direction of the press's force. Based on this assumption calculation formulas are obtained for the stresses and the forces. Experimental data are furnished for pressings using lead samples. A. D. Tomlenov

*Courtesy Referativnyi Zhurnal, USSR*

**Book—3008. Loladze, T. N., Wear of cutting tools [Iznos riezusczego instrumienta]**, Moskva, Maszgiz, 1958, 356 pp. 17 r 60 k.

This book consists of eight chapters, in which the author discusses problems relevant to tool edge wear. In the first four chapters he deals in turn with metallographic test methods of the friction zone; the kind of contact of edge and workpiece; temperature distribution in interfacial layers; and coexistent structural changes in surface layers.

An extensive treatment is given of the factors and conditions affecting geometric changes of tool edge during cutting (Chapter V).

Of great interest are the author's investigations and views on adhesive wear (Chapter VI), and diffusive wear of tool edges (Chapter VII). Author demonstrates that the diffusive processes may be of special importance for tool edge wear for particular cases of cutting. The final part of the book (Chapter VIII) deals with a chemical type of wear of carbide tips.

Author shows practical applications of the theories presented, discussing hot machining (with heated machined layers).

The book is a valuable contribution to the development of wear theory of cutting tool edges. J. Kaczmarek, Poland

**3009. Solaja, V., and Hughes, H. L., Some electrical phenomena in metal cutting,** *Wear* **2**, 4, 311-314 (Brief Notes), May 1959.

## Fracture (Including Fatigue)

(See also Revs. 2929, 2985, 3042, 3359)

**3010. Berry, J. P., Some kinetic considerations of the Griffith criterion for fracture: Part 1, Equations of motion at constant force; Part 2, Equations of motion at constant deformation,** *J. Mech. Phys. Solids* **8**, 3, 194-216, Aug. 1960.

Equations of motion for a propagating crack lead to Griffith-type condition containing a dimensionless parameter $n$ depending on original crack length and rate of straining. Actual Griffith criterion for start of crack propagation then is the minimum (= zero) of kinetic energy term as function of crack length and corresponds to a definite value of $n$. Stress starting crack propagation must slightly exceed Griffith stress which still gives zero crack velocity and acceleration; the excess, characterized by $n$, increases with decreasing initial crack length. Griffith criterion is first satisfied when stress-strain curve of material intersects locus for Griffith criterion, and graphical representation of locus reveals kinetic-to-surface-energy relationship and influence of initial crack length.

Essential differences in loci for tensile and cleavage fracture show that in cleavage the Griffith criterion is less influenced by original crack length than in tension. If strain at start of crack propagation is subsequently held constant, the crack stops after stress has dropped below Griffith locus and acquired kinetic energy is exhausted. For tensile fracture from small initial crack the final crack length depends on position of first point of intersection of stress-strain curve and Griffith locus. For large initial cracks under tension and all cleavage fractures, the final crack length depends only on amount by which Griffith criterion has been exceeded. Quantitative experimental verification of theory is predicted to be best for small initial cracks in tension or large ones in cleavage.

Paper clarifies several doubtful features in present applications of fracture criteria. M. L. Meyer, England

**3011. Sanders, J. L., Jr., On the Griffith-Irwin fracture theory,** *ASME Trans.* **82** E (*J. Appl. Mech.*), 2, 352–353 (Brief Notes), June 1960.

The purpose was to express, in the form of a contour integral, the Griffith-Irwin criterion for crack extension for the case of a crack in a plate where the plane-stress theory applies. Author concludes the path independence of the integral makes it clear that the amount of strain energy available in the cracked structure is irrelevant but that the strength of the square-root singularity at the tip of the crack is critical. M. Holt, USA

**3012. McClintock, F. A., and Sukhatme, S. P., Travelling cracks in elastic materials under longitudinal shear,** *J. Mech. Phys. Solids* **8**, 3, 187–193, Aug. 1960.

Analysis of stress and strain around traveling cracks is easier for longitudinal shear, i.e. anti-plane strain, than for tension. Author shows that dependence of applied stress on crack velocity and the tendency for crack forking above 0.6 $c_2$ approx. are similar for both cases so that simpler shear analysis may also provide insight into tension problem under more complex conditions, e.g. growing cracks or elastic-plastic conditions. Griffith's energy criterion and Neuber's average shear strain criterion are shown to yield similar results in shear analysis.

M. L. Meyer, England

**3013. Yukawa, S., Nonmetallic inclusions and fracture behavior of steels,** *ASME Trans.* **82** D (*J. Basic Engng.*), 2, 411–416, June 1960.

Some observations showing the adverse effects of certain kinds of nonmetallic inclusions on the strength and fracture behavior of alloy steel test specimens are presented. The observations pertain to the effects in tests at low and high temperatures and under steady and cyclic loading. From author's summary

**3014. Pankowski-Fern, Regina, Homes, G. A., and Bauwens, J.-C., A criterion for brittle rupture in ductile metals** (in French), *C. R. Acad. Sci. Paris* **250**, 13, 2368–2370, Mar. 1960.

**3015. Tarasenko, I. I., and Tarasenko, E. N., The conditions governing the brittle stability of isotropic materials** (in Russian), *Zap. Leningrad Gorn. In-ta* **36**, 3, 146–155, 1958; *Ref. Zh. Mekh.* no. 9, 1959, Rev. 10994.

The authors propose to introduce into the generalized critical stress a linear function for the mean (hydrostatic) stresses because of the initial presentation of the case, that the stability in brittle fracture must be linked with the mean spacing between the atoms (ions) which in its turn is determined by the magnitude of the spherical tensor. Examining the case in which the stress tensors' deviators and deformation are similar the authors put forward the following simple formula: $S_1 - \beta(S_2 + S_3) = S_p$, where $S_1$, $S_2$ and $S_3$ are the three principal stresses, $S_p$ the stability during tension, $\beta$ the coefficient equal to the relation of the stability during tension $S_p$ and the stability during compression $S_c$. The applicability of this formula is verified by the authors by reference to data given in the literature for the general case of the plane and three-dimensional problem (the experiments carried out by Karman, Boker, Davidenkov, Stavrogin) and by previous tests by the authors themselves. Satisfactory agreement is obtained, while the relation of the calculated stress to the experimental shows it to be close to 1 (the divergence of this order over a number of percentages, and only in one case did this reach 16%).

N. N. Davidenkov
*Courtesy Referativnyi Zhurnal, USSR*

**3016. Lowengrub, M., Stress in the vicinity of a crack in a thick elastic plate,** AFOSR TN 60-1449 (Duke Univ.), 16 pp., Nov. 1960.

**3017. Widmer, R., and Grant, N. J., The creep-rupture properties of 80 Ni-20 Cr alloys,** *ASME Trans.* **82** D (*J. Basic Engng.*), 4, 829–838, Dec. 1960.

The creep-rupture properties and the creep behavior of a series of 80 Ni-20 Cr alloys were investigated over the 1200 to 1800 F range. In the group were two vacuum-melted and three air-melted alloys. Among these five heats there were small but important differences in minor constituents such as silicon, manganese, sulfur, and possible tramp elements, primarily as a result of the melting practice in each case. Of particular interest in this study was the variation in ductility which the various alloys exhibited in creep-rupture tests which lasted from a few minutes to more than 1000 hr. A close examination of ductility behavior was undertaken by breaking down the creep curve into primary, secondary, and third-stage creep as a function of stress and temperature. It is shown that the stress-rupture properties are not affected over a wide temperature and stress range by a change in minor constituents, whereas the ductility behavior can vary considerably.

From authors' summary

**3018. Garofalo, F., Survey of various special tests used to determine elastic, plastic, and rupture properties of metals at elevated temperatures,** *ASME Trans.* **82** D (*J. Basic Engng.*), 4, 867–880, Dec. 1960.

Testing techniques employed in determining the elastic moduli, that is, Young's modulus, shear modulus, and Poisson's ratio, at room and elevated temperatures are described. These techniques depend on static or dynamic measurements. A comparison and an analysis of test results determined by these two methods are presented. The effect of composition, grain size, and various transformations on the elastic moduli or their temperature dependence is discussed.

A review of techniques and experimental data on the effect of high strain rates on plastic and rupture behavior of metals, and alloys at elevated temperatures is presented. It is shown that recovery effects explain qualitatively the results obtained. A brief description of the various stages of recovery is also presented.

The variation of hardness with temperature is discussed for pure metals and alloys, including a description of a typical hot-hardness tester. The relationship between hardness and tensile strength, creep, and creep-rupture behavior is briefly summarized. The use of the hot-hardness tester as a research tool for following solid-state reactions at elevated temperatures is discussed. These reactions may depend on temperature, time, or plastic strain or a combination of these.

From author's summary

**3019. McEvily, A. J., Jr., The effect of repeated stressing on the behavior of lithium fluoride crystals,** NASA TR R-91, 31 pp., 1961.

Results of reversed-bend tests on LiF single crystals at 2 cpm are similar to those previously obtained at 1800 cpm, indicating that speed effects are relatively unimportant. Reversed-torsion tests yield additional evidence that vacancy formation is not a primary cause of fatigue failure. Since LiF does not readily cross slip, the absence of fatigue failures lends further support to the theory that cross slip is necessary in order to obtain usual fatigue behavior. The results demonstrate that dislocation pileups do not lead to failure in LiF, and that plastic deformation in the course of reversed cycling will relax an initially applied mean load. Etch-pit studies yield visual evidence of the growth of slipbands during cycling. From author's summary

**3020. Investigation of thermal effects on structural fatigue: Part 1,** WADD TR 60-410 (Douglas Aircr. Co., Santa Monica, Calif.), 189 pp., Aug. 1960.

A procedure for design of fatigue-resistant structures under an elevated temperature environment is presented. Constant load amplitude and spectrum fatigue tests were run on notched and unnotched Ph 15-7Mo and AM 355 stainless steel specimens at temperatures up to 800°F. The hydraulic spectrum loader is described. Methods of accounting for nonlinear damage propagation and large preloads are developed. A digital computer program is presented based on these methods. The test spectrum fatigue lives are compared with predictions by cumulative damage analyses. These analyses include Miner's linear damage theory as well as several nonlinear damage theories.

From summary

**3021. Yokobori, T., Stress criterion for fatigue fracture of steels,** *J. Mech. Phys. Solids* **8**, 2, 81-86, May 1960.

An approach is presented to the problem of correlating the macroscopic fatigue strength of steels with microscopic explanations of present dislocation theory. The stress concentration or notch effect due to the obstacle itself against which the dislocations pile up is taken into account, in addition to assuming that stress concentration at the piled-up dislocations plays an important role in fatigue fracture of steels. The agreement with the characteristic features in stress criterion for fatigue fracture of steels is good.

From author's summary by F. Garofalo, USA

**3022. Schreiber, H.-H., and Ulsenheimer, G., Fatigue of antifriction bearings** (in German), *Wear* **3**, 2, 122-143, Mar./Apr. 1960.

A review is given of investigations to date on pitting of antifriction bearings and of resulting hypotheses in order to obtain what might be called a general view on the whole problem. A discussion would help to clarify the situation and furnish new aspects for further experimental work.

The study deals first with pitting as a phenomenon of strength in contrast to phenomena of wear. Experimental results are then individually grouped under the following headings: (1) effects of material treatment and processing; (2) stress distribution caused by external loads; (3) effect of lubricant; (4) metallurgical phenomena.

Two methods for devising a life theory for antifriction bearings are presented, the first based on empirical data. Extensive investigations of the various influences will still be required, however, to arrive at sufficiently exact statements. The starting point for the second method is the knowledge about the process of fatigue and its underlying causes. Here, metallurgy will have to work out a basis for further fatigue life studies.

From authors' summary by H. F. K. Drescher, Germany

**3023. Farkas, J., A new method for the determination of fatigue strength of welded joints** (in Hungarian), *Gép* **12**, 1, 35-36, Jan. 1960.

Another formula is presented for the expression of fatigue strength of welded joints, a simplification over Stüssi and Newmann's approach. The interpretation is almost impossible for those not familiar with Hungarian standard symbols; a legend would be a great improvement.

D. D. Vasarhelyi, USA

**3024. Vagapov, R. D., Khripina, L. A., and Shishorina, O. I., Assessment of fatigue strength of heavy machinery components by means of the results of tests on scale-model samples** (in Russian), *Izv. Akad. Nauk SSSR, Otd. Tekhn. Nauk* no. 7, 15-23, 1958; *Ref. Zh. Mekh.* no. 9, 1959, Rev. 10913.

The problem of modelling in combination with the determination of the carrying capacity of heavy machinery components in relation to their form and dimensions is investigated by way of an example on a rotor construction in its application to places showing sharp changes in sections. It is demonstrated that designing the models and evaluating the results of the tests on them has to be based on the principles of fatigue collapse. In reaching this conclusion the assumption is made that the distribution of the stresses close to the limit of fatigue $\sigma_w$ is very near to the elastic, while the magnitude of $\sigma_w$ is expressed through the magnitude of the first principal stress. A model is proposed for the evaluation of the fatigue strength, which reproduces the conditions of loading in the surface layer of the most heavily stressed of its components. The designing of the model and the selection of the form of its loading for the purpose of checking experimentally the procedure to be used were carried out as suitable to a shaft with a sufficiently deep annular groove. The experimental check of the procedure was made on samples cut from a rolled shaft with a diameter of 110 mm. The material of the ingots of St 45, all of one smelt, was passed as standard and this enabled the possibility to be verified of calculating the carrying capacity on the assumption of the elastic state setting in at a comparatively high plasticity of the steel. Five series of samples with a diameter of 50 mm with radii of the notches of 11, 7.5, 3.5, 1 and 0.5 mm and geometrically similar models of 7.5 mm were tested for variable deflection during rotation. The experimental check confirmed the possibility of evaluating the carrying capacity on the lower boundary of dissipation on the basis of small-sized models which would reproduce the conditions of loading and resistance of the metal in the surface layer.

L. K. Gordienko
*Courtesy Referativnyi Zhurnal, USSR*

**3025. Makogonov, V. E., and Terminasov, Yu. S., An x-ray investigation of the mechanism of fatigue in metals having different sized crystals** (in Russian), *Trudl Leningrad Inzh.-Ekon. In-ta* no. 23, 46-67, 1958; *Ref. Zh. Mekh.* no. 9, 1959, Rev. 11002.

Authors tested samples of pure aluminum, electrolytic copper and technical iron (Armco iron). The samples were prepared with different sizes of crystals in accordance with the recorded mechanical and thermal processes to be found in the literature. Tests were carried out for symmetrical deflection of cantilever specimens with a fixed magnitude for the amplitude of the deflection at different values for the amplitude. A description is given of the samples and testing apparatus for use with frequencies of from 0 to 1400 revs/min. The procedure adopted for the x-ray photography and treatment of the results is given in detail. From a study of the results of the x-ray investigation of the atomic crystalline lattice of aluminum the authors conclude that the criteria for fatigue regarding the change in intensity in the interference lines, which was taken to be true only for preliminarily annealed carbon steels, could find more universal application.

N. G. Kushelev
*Courtesy Referativnyi Zhurnal, USSR*

**3026. Jaumotte, A., Kiedrzynski, A., and Strebelle, J., Machine for fatigue testing of a cable in tension** (in French), *Rev. Tijdschr.-Mecan. Werk* **6**, 3, 137-144, 1960.

**3027. Mathieson, R., and Hobbs, J. M., Cavitation erosion: comparative tests,** *Engineering* **189**, 4892, 136-137, Jan. 1960.

Cavitation erosion tests were made on a number of different materials. The results were correlated in an effort to compare the erosion resistance properties of the various metals, the effects of the testing method, and mechanical, chemical, and electrolytic effects.

Two methods of testing were used: (1) the drop impact test, in which specimens were rotated through two jets of liquid under a 70-foot head, (2) flow tests in a small water tunnel with throat speeds up to 100 feet per second. The flow tests most closely simulate actual cavitation conditions but require considerable test time to show appreciable effects. Correlation between the two

test methods is shown in a summary table, in which aluminum bronze, selected as a standard, is given a value of unity. The general belief that hardness and strength of a material is the best guide to its erosion resistance was supported by the results. There was good correlation in order of the effects from the two methods of testing although actual magnitudes varied considerably from metal to metal.

A great deal of useful material has been accumulated; detailed data are available upon request to the authors.

E. G. Allen, USA

3028. **Lelong, P., Moisan, J., and Herenguel, J., Observations of the nature of the corrosive attack on aluminum-iron-nickel alloys in steam above the critical temperature** (in French), *C. R. Acad. Sci. Paris* **250**, 2, 340–342, Jan. 1960.

3029. **Wilson, R. G., Peters, R. W., and McEvily, A. J., Jr., A study of the oxidation of materials in an arc-image furnace,** NASA TN D-644, 26 pp., Jan. 1961.

A series of tests on silicon carbide, graphite, nylon, Bakelite, Fiberglas phenolic, and ammonium chloride has been conducted in an arc-image furnace, with radiant heat fluxes at the specimen surfaces ranging from 100 to 1000 Btu/ft$^2$-sec. The effects of radiant heat flux and concentration of oxygen in the specimen environment on the rate of material loss were determined. It was found that for the upper half of the heat-flux range the rate-controlling process for materials that oxidize was the diffusion rate of oxygen to the specimen surface.

From authors' summary

## Experimental Stress Analysis

(See also Revs. 2858, 2999, 3164)

3030. **Waling, J. L., and Greszczuk, L. B., Experiments with thin-shell structural models,** *J. Amer. Concr. Inst.* **32**, 4, 413–432, Oct. 1960.

Load-deflection measurements were made on a hyperbolic paraboloidal model made from styrofoam reinforced with wires. Data on styrofoam structural properties, and on the load-deflection results, are reported. Photographs of other tests on complex models are accompanied by brief descriptions. The purpose of the article is to demonstrate the utility of plastics in the fabrication of concrete structures.

H. Becker, USA

**Book—3031. Jessop, H. T., and Harris, F. C., Photoelasticity; principles and methods,** New York, Dover Publications, Inc., 1960, viii + 184 pp. $2.00. (Paperbound)

This new Dover edition is an unabridged and unaltered republication of the first edition published in 1949.

Ed.

3032. **Robertson, G., Stress analysis of flanged pipe joints by the "frozen stress" photoelastic method,** *J. Mech. Engng. Sci.* **2**, 3, 254–272, Sept. 1960.

Author describes investigation to determine elastic distribution of stress in rings and flanges of different thicknesses loaded uniformly round inner edge and round diameter corresponding to the pitch circle. Influence of flange thickness, number of bolt holes and length of hub, was determined on model flanges subjected to simulated bolt loading and on flanges integral with short length of pipe and subjected to internal pressure. Complete distributions of tangential and radial contour stresses were obtained over free surface of rings and at both types of radial section of symmetry in flanges, and axial deflections were also measured. Test results are compared with those computed from various theoretical solutions and good agreement is generally found.

G. G. Meyerhof, Canada

3033. **Schwieger, H., and Reimann, V., Photoelastic investigation of a transverse impact on a circular plate** (in German), *ZAMM* **39**, 5/6, 198–213, May/June 1959.

The behavior of a simply supported circular plate under the impact of a falling mass is studied photoelastically. Two plates of identical glass are cemented together with a layer of reflecting material between them. When the plate is deflected, a plane polarized light beam after twice traversing one half of the plate experiences a relative retardation which is proportional to the maximum twisting moment. Measurements are carried out point by point by means of a special polariscope which consists of the polarizer, a quarter-wave plate, the analyzer and a photocell. The photocell is connected to an oscilloscope through an amplifier. The plane of polarization is set at 45° with respect to the planes of the principal bending moments.

Two measurements are carried out at each point, corresponding to two different positions of the axis of the quarter-wave plate: (a) axis parallel to the plane of polarization; (b) axis at 45° with respect to the plane of polarization. Both measurements are registered on a film. From these data the tangent of the relative retardation is computed. From the relative retardation the maximum twisting moment is obtained. By numerical integration of the twisting moments, deflections and principal bending moments are calculated. Maximum transversal force originated during the impact time is compared with theoretical values. Theoretical impact time is shorter than the experimental time and the theoretical impact force larger than the experimental force. Deflections at the center point of the plate were also measured. Measured values and values obtained by integration are in fair agreement.

C. A. Sciammarella, Argentina

3034. **Carmichael, A. J., and Douglas, A. G., Gelatin solution as a model and photoelastic material,** *Austral. J. Appl. Sci.* **11**, 2, 272–288, June 1960.

Aqueous solutions of gelatin, when set, behave in a manner which may be described as "rubber-like." This material has good elastic properties and remarkable optical sensitivity, which makes it an ideal photoelastic material for representing body-force problems and problems involving large elastic deformations.

This paper describes important mechanical and optical properties of various gelatin mixes, and the effect of including glycerin. The behavior of gelatin in torsion, tension, and compression, and the mechanical creep and relaxation are considered. The variation of these properties with time, temperature, and pH is discussed and sufficient information is given to enable the confident construction of models.

The birefringence of gelatin solution has been known for some time, but it is unfortunate that very little detailed investigation of this phenomenon has been reported since the pioneer work of Farquharson and Hennes (1940). The present paper outlines an accurate calibration of this material, together with a discussion of the variation of optical sensitivity with time and temperature.

From authors' summary

No mention is made in the paper about the possibilities of using gelatin for three-dimensional work by the stress-freezing method. Work of this nature has already been done in Spain by Toroja of the Laboratory of Civil Engineering in Madrid, and a paper describing the study was published in France by G.A.M.A.C. in 1952. This appears important for a complete analysis of the behavior of the material described.

F. Zandman, USA

3035. **Lagarde, A., On the problem of the photoelastic behavior of a viscoelastic material in dynamic excitation** (in French), *C. R. Acad. Sci. Paris* **251**, 5, 633–635, Aug. 1960.

**3036. Lagarde, A., Application of vibrations in measuring the mechanical and photoelastic properties of plastic materials** (in French), *C. R. Acad. Sci. Paris* **250**, 16, 2796–2798, Apr. 1960.

**3037. Lagarde, A., and Jacquesson, R., Measuring of the mechanical and photoelastic characteristics of a plastic material under sinusoidal forces of low frequency** (in French), *C. R. Acad. Sci. Paris* **250**, 6, 969–971, Feb. 1960.

**3038. Sapaly, J., On an improvement of the procedures in photoelastic measurement** (in French), *C. R. Acad. Sci. Paris* **250**, 2, 287–289, Jan. 1960.

**3039. Bloss, R. L., Evaluation of resistance strain gages at elevated temperatures,** *Materials Res. & Standards* **1**, 1, 9–15, Jan. 1961.

To satisfy the need for strain measurements at elevated temperatures a number of types of "high-temperature" strain gages are available. A comprehensive evaluation of these gages is needed because of the lack of available information on their performance and because of difficulty that has been encountered in their use. A facility for conducting such an evaluation is described. Typical results obtained during evaluation tests are given to show the capability of the equipment, to point out some of the problems encountered in the use of these gages, and to illustrate the need for comprehensive gage evaluation prior to use. The results shown are only indicative of the performance of the particular gage tested. Significantly different results might be obtained from other gage types.

From author's summary

**3040. Sukhomel, E. G., The rational disposition of strain gauges for the determination of the deflection moments in beams of rectangular section** (in Russian), *Trudi Kievsk. In-ta Inzh. Vodn. Kh-va* no. 7, 277–288, 1957; *Ref. Zh. Mekh.* no. 9, 1959, Rev. 11077.

Study was made of the rational disposition of strain gages on the contour of a rectangular transverse section of a beam for a case of oblique deflection, based on the condition of minimum error in the computation of the deflection moments acting on that beam. For the evaluation of the errors the values of the full quadratic mistakes are utilized. The magnitudes of the calculation errors determined by the obtained formulas are compared with the calculation errors of the experiment which were found with consideration for the errors in measuring the load forces and the coordinates of the strain gage transmitters. Some improvements relating to precision are introduced in the previously-made recommendations (the results of work done) and the point is stressed that with an oblique (relative to the principal axes of inertia of the transverse section of the beam) disposition of the strain gages infinitely large errors in measurement are theoretically possible. It is shown that in order to ensure minimum errors in the determination of the moments the strain gages have to be disposed at maximum distances (on different sides) from one of the principal axes of inertia of the transverse section of the beam and at the same time at identical distances (but imperatively on one side) from the other principal axis of inertia.

V. A. Radzievskii

*Courtesy Referativnyi Zhurnal, USSR*

**3041. Onogi, S., Keedy, D. A., and Stein, R. S., The measurement of dynamic birefringence,** AFOSR 61-28, (Univ. Mass., Dept. Chem. TR1), 2 pp. + figs., Jan. 1961.

**3042. Redshaw, S. C., and Rushton, K. R., An electrical analogue solution for the stresses near a crack or hole in a flat plate,** *J. Mech. Phys. Solids* **8**, 3, 173–186, Aug. 1960.

An electrical analog for solving the biharmonic equation was used to determine the elastic stresses near a hole or crack in a flat rectangular plate subjected to uniaxial tension. Problems concerning both simply and multiply connected regions were solved by the method and, where a comparison between experimental results and existing analytical solutions was possible, the agreement was good.

The advantages and limitations of the method are discussed and the desirability of extending the method for the determination of the stresses after plastic yielding has occurred is emphasized.

From authors' summary by B. Paul, USA

**3043. Belyaev, V. A., The impact of a solid object on the blade of a propeller screw** (in Russian), *Trudi Gorkovsk. Politekhn. In-ta* **14**, 1, 17–22, 1958; *Ref. Zh. Mekh.* no. 9, 1959, Rev. 10818.

A description is given of the apparatus used and of the test employed to determine the stresses produced in the basal section of the blade. Both the shielded and unshielded systems are investigated. The results of the experiments are compared with calculated results. The author explains the divergence in one case by the increase of the stiffness of the rubber when under dynamic load, and in the other—by the presence of pliable supports.

V. K. Egupov

*Courtesy Referativnyi Zhurnal, USSR*

**3044. Ulegin, V. G., Accuracy of machines for testing tension and compression in metals,** Measurement Techniques no. 8, 604–606, July 1960. (Translation of *Izmeritel'naya Tekhnika, USSR* no. 8, 22–23, Aug. 1959 by Instrument Society of America, Pittsburgh 22, Pa.)

Author observes that elastic properties of autographic testing machine influence the apparent characteristics of a test specimen as recorded on chart. Careful tests on three specific instruments cause author to conclude that "existing machines with stirrup links cannot provide, under certain conditions of measurement, errors below ±1% of the measured force." Since the "certain conditions" quoted are not specifically defined, reviewer feels that quantitative aspects of paper are of questionable value to readers.

B. Paul, USA

**3045. Schumann, W., Experimental determination of three-dimensional stresses** (in German), Publ. du Laboratoire de Photoelasticite, Ecole Polytechnique Federale, Zurich, no. 8, 60 pp., 1959.

A good summary of the three-dimensional photoelasticity methods. The author contributes the method of hexagonal cuts, an elimination process of the effect of large deflections and the utilization of the Moire effect; his originality in compiling his method of analysis should be recognized. A more careful reference to original sources would be an improvement. The basic principle of elimination of the effect of large deflections in model method is to be found in W. J. Eney's publication in *Proceedings, Society of Experimental Stress Analysis* **VI**, no. 2, 1949. The photographic method to obtain trajectories from isoclinics was first published by D. Vasarhelyi, *Osterreichisches Ingenieur-Archiv,* **2**, no. 1, 1947. Recognition of these priorities would not reduce the value of the author's original contributions.

D. D. Vasarhelyi, USA

## Material Test Techniques

(See Revs. 2852, 2985, 2999)

## Properties of Engineering Materials

(See Revs. 2918, 2922, 2923, 3011, 3021, 3027, 3028, 3029)

## Structures: Simple

(See also Revs. 2909, 2912, 2937, 2951, 2953, 2968, 3030)

**Book—3046. Bassin, M. G., and Brodsky, S. M., Statics and strength of materials,** New York, McGraw-Hill Book Company, Inc., 1960, viii + 354 pp. $7.

This book, based on "Practical mechanics and strength of materials" by C. W. Leigh and J. F. Mangold, is claimed by the authors to retain a practical approach but to introduce modern practice so as to attain increased effectiveness as both a learning and a teaching aid. The text, the majority of which has been completely rewritten, is directed toward community college, technical institute, and junior college programs not requiring the use of calculus.

Reviewer considers the first six chapters give an introduction to statics and Chaps. 7-16 a brief insight into some of the field of strength of materials and elementary stressing. Each chapter ends with a collection of problems relevant to the text, some answers being given in the back of the book. Tables are given, in an appendix, of those data necessary to solving the problems; these include typical mechanical properties of some metals, designing properties of some structural sections, etc., and finally a table of the more important formulas with textual identification.

This book gives a very simplified treatment of its subject and the elimination of calculus leads in many cases to a roundabout proof which often obscures the picture of the physical processes involved. Having proved a formula the text proceeds to demonstrate its use in sample problems, upon which it relies heavily; reviewer estimates that more than half the book is the working out of demonstration problems.

Reviewer is critical of the lack of precision displayed in some definitions and uses of terms. For instance $E$ is used in the proof of proportionality between shear stress and strain in torsion, later to be changed to $G$, and in the same chapter the relation of Hooke's law is used but this is the only reference to the law that the reviewer can find in the whole book, all most confusing.

To sum up, this is a book that might be of use to people doing very elementary studies in the subject, who have a severely limited mathematical background; certainly it is not the kind of book that would be of use to the professional engineer as a reference to formulas or methods of attack.

A. F. W. Langford, Australia

**Book—3047. Carpenter, S. T., Structural mechanics,** New York, John Wiley & Sons, Inc. 1960, xi + 538 pp. $9.50.

In this book are collected the lectures given by the author, Professor of Civil Engineering at Swarthmore College. The first chapters are devoted to the general questions and to ideas which author presupposes already known by the student from previous courses. In the second chapter there is a short exposition of the theory of inflected beams (with a hint about trigonometric series and finite differences) and of applied points of view of the most important theorems regarding the work of deformation. The fourth chapter is devoted to the method of iterative balancing of moments (Cross's method); the study of frames with sidesway is brought again to the solution of a system of equations in which the number of these is equal to the number of the frame floors. A short exposition of the "Deflection method" (Chapter five) follows Cross's method. Special attention (Chapter six) is given to the simplifications which are obtained in considering the reactions applied to the elastic center of a structure. The seventh chapter deals with the equilibrium of cables, with some discussion of suspension bridges. References to the vibrations of structures and to axially loaded members conclude the volume.

All chapters follow in logical progression; the exposition is simple and clear. Examples and exercises with answers are given so that the student can insure his understanding of the problems. At the end of every chapter there are brief bibliographic references limited to works in the English language.

P. Pozzati, Italy

**3048. Crichlow, W. J., and Haggenmacher, G. W., The analysis of redundant structures by the use of high-speed digital computers,** *J. Aerospace Sci.* **27**, 8, 595-606, 614, Aug. 1960.

A matrix formulation of analysis of statically indeterminate structures using the skin-stringer-beam model is presented. Beyond a clearly written review of basic equations and matrix formulation, the paper describes useful techniques for reducing the effort of programming and increasing accuracy. Emphasis is placed on a concept of providing a set of $n$-independent minimum-sized self-equilibrating internal load systems replacing the concept of $n$-redundant elements. Engineering and computational advantages arise, particularly in large-scale applications.

An interesting scheme for providing for cut-outs by adding by-pass elements of equal negative flexibility is described.

A number of examples illustrate special features and demonstrate the versatility of the approach in analysis of large-scale complex structures.

Reviewer believes paper to be timely and to be of great interest to engineers desiring to take advantage of high-speed computer facilities.

J. A. Cheney, USA

**3049. Livesley, R. K., Optimum design of structural frames for alternative systems of loading,** *Civ. Engng., Lond.* **54**, 636, 737-740, June 1959.

Author presents a clear graphical representation of the problem of least-weight design of rigid jointed frames under alternate loading conditions, using theory of plastic collapse. Load system may vary between prescribed limits, and is represented (for the two-load system considered) by a "load diagram." Possible failure mechanisms are represented on a "design diagram." For each mechanism a limiting value is found for which design is safe (i.e. on point of collapse) under all load combinations. Assuming weight to be proportional to the fully plastic moment, lines of constant weight are drawn on the "design diagram." The solution is the line that touches the safe region defined by all possible mechanisms.

Extension to more loading systems and more plastic moments can be made by linear programming, requiring use of a digital computer. Approach is useful in obtaining a general picture of structural behavior. Minimum weight design is one that most successfully balances effect of a number of extreme loading cases.

S. J. Fenves, USA

**3050. Stolle, H. W., Analysis of frameworks whose beams are elastically supported** (in German), *Bautechnik* **37**, 5, 185-191, May 1960.

Article is based on the well-known framework analysis which is extended to include structures whose beams, or some of them, are elastically supported. This elastical supporting changes the generally used factors in the framework calculations. Therefore these factors are tabulated for the general cases when the load is applied in the plane of the structure. The analysis is illustrated by two examples.

The given method can be extended to cover spatial frameworks.

K. Angervo, Finland

**3051. Rusch, H., Researches toward a general flexural theory for structural concrete,** *J. Amer. Concr. Inst.* **32**, 1, 1-28, July 1960.

This is one of the few contributions which may lead to the evolution of a new and basically different approach in the design of concrete structures. So far several theories have been formulated

which start from different, and at times contradictory, assumptions in dealing with flexure problems. According to this author, only tests of over-reinforced members can furnish a true measure of validity of flexure theory. As such there is need for a theory which is not restricted to approximate results in limited range, and which is furthermore based on the actual properties of materials and is valid for all cases of loading, from pure bending to pure compression. This is what the author develops here, covering such topics as rate of loading effects, stress-strain relations in flexure, compression strain in extreme fibers, and the effects of position of the neutral axis and shape of cross section on ultimate strain. Strength of concrete decreases under the action of sustained loads, while creep of concrete leads to an increase in concrete strain in the extreme compressive fiber. These aspects have been woven into a set of practically usable curves which cover the entire range of stress conditions from pure bending to concentric compression.

Presented in a masterly fashion the author has set in this paper new standards of flexure design which may well become, after long-term applications, acceptable for all types of structural concrete usage, regardless of concrete quality and the type of steel incorporated and independent of whether prestressing is applied or not. S. K. Ghaswala, India

**3052. Bauer, F., Calculation of bearing moments of prestressed concrete cross sections** (in German), *Zement u. Beton* no. 18, 11-16, Apr. 1960.

Author calculates the stresses occurring in a prestressed reinforced concrete beam of rectangular cross section for the case in which a parabolic variation is admitted for the stress-strain diagram of the concrete. Theory is extended to the case in which the cross section may be resolved into rectangles (e.g., case of a T-shaped cross section).

Results applicable in engineering design are plotted in diagrams and used for several numerical examples.

P. P. Teodorescu, Roumania

**3053. Madsen, G., and Biggs, D., Building for economy with hyperbolic paraboloids,** *J. Amer. Concr. Inst.* **32**, 4, 373-383, Oct. 1960.

Economics and fabrication of a hyperbolic paraboloid roof are described. Emphasis is upon economic characteristics of this type of construction. H. Becker, USA

**3054. Harrenstien, H. P., Hyperbolic paraboloidal umbrella shells under vertical loads,** *J. Amer. Concr. Inst.* **32**, 4, 385-401, Oct. 1960.

Experimental stress analysis was made of a hyperbolic paraboloidal umbrella shell using strain gages on a reinforced-concrete model. Strains were measured and load-stress contours constructed for concentrated loads applied parallel to the umbrella axis. The principal feature is a set of nine contour charts showing the stress distributions for loads applied at several positions on the shell. H. Becker, USA

**3055. Tedesko, A., Shell at Denver—Hyperbolic paraboloidal structure of wide span,** *J. Amer. Concr. Inst.* **32**, 4, 403-412, Oct. 1960.

Design features and fabrication details are presented for a free-spanning reinforced-concrete shell hinge-supported at each of the four corners of the rectangular planform. Emphasis is upon fabrication procedures. H. Becker, USA

**3056. Fischer, L., Determination of membrane stresses in elliptic paraboloids using polynomials,** *J. Amer. Concr. Inst.* **32**, 4, 433-441, Oct. 1960.

Equations are developed for stresses in elliptic paraboloids, employing a polynomial form of a stress function. The analysis is limited to determination of membrane stresses. Bending information is not included. A discussion of the limitations of the method concludes the paper. H. Becker, USA

**3057. Hangan, M., The order of occurrence and the rotation of plastic hinges for a static structure subjected to a permanent loading and a group of proportionally increasing loads** (in Roumanian), *Studii Si Cercetari Mecan. Appl.* **11**, 6, 1445-1457, 1960.

Paper represents a continuation of a previous study on hyperstatic structures in the plastic range [AMR **13**(1960), Rev. 2302] and includes the influence of a permanent loading besides the influence of proportionally increasing loadings. The order of occurrence of plastic hinges, the load to which the structure is subjected when various plastic hinges occur as well as the successive rotations of various plastic hinges are determined.

An example of application for continuous beams, interesting for engineering design, is included.

P. P. Teodorescu, Roumania

**3058. Sahmel, P., Analysis of frames with elastic tension members by an extension of Kani's iteration method** (in German), *Bauingenieur* **35**, 7, 263-268, July 1960.

Author states that Kani's iterational method is popular for the design of multi-storey frames although he considered only special case, e.g. bents having rectangular joints and rigid supports. Author shows extension of Kani's method for a more general case, e.g. bents with tie-rods and bents having nonrectangular joints. For that purpose author develops five formulas which are based on Kani's method and on deformation of the structure.

Using these five formulas, author elucidates development of special-case formulas for some practical forms of bents. Detailed numerical solutions are shown for four practical problems. As each special case requires basic algebraic solutions, the method may be regarded as semi-analytical and semi-iterational. Although the analytical calculations required are not complicated, some practical engineers may object to marring the clarity of the iterational methods with some tedious algebraic equations.

Reviewer believes that the applicability of the method is a matter of personal judgment, depending on the taste of practical engineer. E. Ben-Zvi, Israel

**3059. Halasz, O., Roller, B., and Vertes, G., Calculation of suspended masses by linear theory** (in Hungarian), *Mélyépítéstudományi Szemle* **10**, 3, 131-137, Mar. 1960.

Authors investigate the relation between the form of the suspended masses and the tensioning forces of the ropes. They present a method for the determination of the form of the mass if the tensioning forces are given. The distribution and the magnitude of the tensioning forces can be chosen arbitrarily, but the edge girders must be closed.

The presented method is based on the relaxation method. It consists of removing the ropes in every knot and then of the balancing of the forces.

A numerical example is given where the ropes—in horizontal projection—are rectangular.

S. Sarkadi Szabo, Hungary

**3060. Bolotin, V. V., Application of statistical methods for evaluation of the strength of structures subjected to seismic effects** (in Russian), *Inzhener. Sbornik Akad. Nauk SSSR* **27**, 58-69, 1960.

The theory of random processes is applied to the analysis of the response of structures subjected to earthquakes. The assumptions are made that the structure is a linear mechanical system and the

earthquake is nonstationary random. The problem of safety of such structures is also discussed from the statistical point of view.

M. P. Bieniek, USA

**3061. Nazarov, A. G., The theory of modelling in its application to building construction and earthquake-proof structures** (in Russian), The Joint Scientific Session of the Building Materials and Construction Institutes of Transcaucasia, Rep., 1957, Baku, 1958, 22–30; *Ref. Zh. Mekh.* no. 9, 1959, Rev. 10594.

Methods are given for the approximate investigations of building constructions as regards stability to seismic disturbance by means of tests on models prepared in accordance with the principles of modelling. Two methods of approach are recommended: (1) the production of oscillations in the soil base by the simplest principle not reflecting the actual principle of oscillations in earthquakes, together with the testing of two model buildings one of which is selected as the standard, (2) an exact reproduction of the oscillations of the base in an earthquake with testing of the model under observation. In the first case the result obtained is a relative evaluation of the reaction to an earthquake; in the second, an absolute record. The first method is considered more suitable for practical realization.

K. S. Zavriev

*Courtesy Referativnyi Zhurnal, USSR*

## Structures: Composite

(See also Revs. 2928, 2929, 2956, 2961, 3060, 3164, 3274, 3330, 3358, 3368)

**3062. Egorov, A. V., Determination of stability of structures** (in Russian), *Gidrotekh. Stroit.* **30**, 2, 25–29, Feb. 1960.

Determination of the safety factor against horizontal sliding and overturning of dams and retaining walls is discussed. The writer recommends using differentiated factors of safety depending upon the accuracy with which acting forces are determined; for example since water pressure can be calculated more exactly than the earth pressure, the factor of safety against water action must be smaller. Since actual acting forces are not exactly equal to those calculated, the possible deviation of actual forces from computed must be taken into account by coefficients, which must be greater than one for pushing and less than one for retaining forces. These corrected values must enter into the formula for the safety factor. Since coefficients are introduced for each force separately, the over-all safety factor can be accepted as equal to 1.0–1.10.

Reviewer believes that the approach of the author is correct, however, it is not entirely new.

B. S. Browzin, USA

**3063. Priscu, R., and Iamandi, C., Considerations on the design of heavy dams of trapezoidal profile** (in German), *Rev. Mécan. Appl.* **5**, 6, 785–795, 1960.

Authors establish some relations leading to an optimum profile from the static and economic viewpoint for heavy dams of trapezoidal profile placed on rocky soils. The results obtained are compared to those for dams with triangular profiles.

P. P. Teodorescu, Roumania

**Book—3064. Calculation procedures for determining the strength of cargo-carrying vessels** (in Russian), (Trudi Tsentr. Nauk-i. In-ta Morsk. Flota, Vol. 17), Leningrad, Moscow Transport, 1958, 128 pp. + illus. 8 r 25 k; *Ref. Zh. Mekh.* no. 9, 1959, Rev. 10823.

The book gives a progressive account, at a contemporary level, of plans for the basic calculations for strength which are carried out when designing the hull. In the first chapter questions are investigated dealing with the drawing of the curve for weight loading and the determination of the deflection moment in smooth water and when statically subjected to wave action. Suitable approximate formulas are found for the determination of the general deflection moment in the midship section and for the calculation value of the intersecting force, which give sufficiently accurate results. The second chapter deals with the details of calculations for general strength. Auxiliary tables are furnished for the calculations of various types of decking for flexure, thus reducing the volume of computation work appreciably. An approximate method for the calculation of coverings is given for the case where the walls of the longitudinal beams of the hull are going over to the elastic-plastic stage of working. In the third chapter a scheme is developed to cover the main calculations for the local strength of dry-cargo ships, while the fourth chapter deals with tankers.

The work being reviewed appears to be the first step to introduce the computational method in marine freighter construction, which up to present times has rested on the rules and regulations of different companies interested in their classification, while in the U.S.S.R. the matter is regulated by the rules of the Marine Register of the U.S.S.R.

A. A. Kurdyumov

*Courtesy Referativnyi Zhurnal, USSR*

**3065. Suhara, J., Three-dimensional theory of the strength of ship hull taking into account the mutual interactions between longitudinal and transverse members,** *Mem. Fac. Engng., Kyushu Univ.* **19**, 4, 229–293, Mar. 1960.

A prismatic section of hull is idealized into a rectangular space framework for which the slope-deflection equations are set up in matrix form suitable for a computer. By making simplifying assumptions, the matrix orders are reduced sufficiently for the solution of some practical examples of still-water loading. The relieving effects of interaction can be large for the transverse frames. Pillars at hatch corners are found to be beneficial. The formulation of the problem as well as the numerical examples should be of interest to engineers concerned with such matters.

J. M. Frankland, USA

**Book—3066. Shanley, F. R., Weight-strength analysis of aircraft structures,** New York, Dover Publications, Inc., 1960, xiii + 404 pp. $2.45. (Paperbound)

This new Dover edition is an unabridged republication of the 1952 edition to which has been added a new preface and bibliographies on optimum design of structures and creep buckling.

From author's summary

## Machine Elements and Machine Design

(See also Revs. 2834, 2978)

**3067. Kobrinskij, A. E., On the selection of the law of motion of a follower, in a cam mechanism** (in German), *Maschinenbautechnik* **9**, 2, 93–98, Feb. 1960.

This paper is the report of a contribution made by the author in a Russian publication. Author states that, in order to avoid hammering in cam mechanisms, the movement of the follower must fulfill certain conditions, namely, the velocity and acceleration diagrams should not show jumps; the values of velocity and acceleration should be zero at the extremities of the displacement period. Author's aim is to show that a family of curves deduced from the generalization of series curves satisfies these conditions.

Sine curves are first considered; their coordinates $v$ and $u$ are transformed into rectangular ones: $x$ and $y$. In these latter values appear the angle $\alpha$ made by the $v$ and $x$ axis, the angle $\lambda$ formed by the asix $v$ and $u$, the amplitude factor $A$ of the sine curve and

its period $K$ measured along the $v$ axis. The angle $\alpha$ is then calculated under the condition that the velocity should be zero at the ends of the period; the accelerations then also become zero at those ends and the $v$ axis becomes the hypothenuse of the triangle constructed with the displacement $L$ and the stroke $H$. The variation of the angle $\lambda$ between definite limits leads, for imposed values of $L$ and $H$, to a whole family of curves satisfying the imposed conditions. Some characteristics of the family are then examined: the value of $A$ (there is an error in equation (14)), the limits between which $\lambda$ ought to be chosen, the maximum value of the acceleration and the special cases where $\lambda = \frac{\pi}{2}$ and $tg\lambda = \frac{L}{H}$.

Curves composed of two sine curves having same tangent and same curvature at their meeting point $M$ are then considered. The velocity and acceleration diagrams are discussed and the influence of the modification in the position of $M$ is shown. A method is given for drawing composed curves.

D. De Meulemeester, Belgium

**3068. Severn, R. T., A refinement in the calculation of the characteristics of a shrink-fitted assembly loaded in torsion,** *J. Mech. Engng. Sci.* **1**, 1, 16–18, June 1959.

The paper presents a recalculation of the numerical examples from two earlier papers, this time with better accuracy obtained by employing a finer mesh (see AMR **12**(1959), Rev. 5932). Main part is the repetition of the above-mentioned papers. Reviewer believes an experimental corroboration might be revealing.

Z. W. Dybczak, Canada

**3069. Walther, R., and Wagenzink, G., Application of a four-bar mechanism for guiding a tool approximately along a straight line** (in German), *Maschinenbautechnik* **9**, 2, 91–93, Feb. 1960.

Author considers the well-known Watt's mechanism. The formula giving the ordinates of the lemniscale curve is established as function of the dimensions of the mechanism. The deviations from the straight line are calculated for a numerical example and the influence, upon these deviations, of a given tolerance in execution is evaluated.

The practical application to a lathe for ceramic material is described. The advantages over arrangements using sliders are developed, especially when small forces are in question and when dust is produced. Author thinks that straight line mechanisms of high precision, with their deviation, should be studied and the results published.

D. De Meulemeester, Belgium

**3070. Zill, F., The development of exact equations for the velocity-time law and the accelerative-time law of the slider in a centrated slide crank mechanism** (in German), *Maschinenbautechnik* **9**, 2, 99–102, Feb. 1960.

These equations are deduced from the usual exact relation giving the values of the slider displacements $s$ as a function of time. The angular velocity $\omega$ is supposed constant and the two terms of the $s$ equation are differentiated twice with respect to time. A numerical example is treated; the values of the angle $\alpha$ which give the maximum velocity for the slider, and the maximum values of the velocity are calculated using the exact equations. These maximum values are compared with the maximum values of the velocity deduced from the approximate equations generally used; the relative error is found very small.

For the accelerations it is shown that the maximum values deduced from the exact and from the usual approximate equations occur for the same angles ($\alpha = 0$, $\alpha = 180°$) and have the same values.

D. De Meulemeester, Belgium

# Fastening and Joining Methods

(See Rev. 3023)

# Rheology

(See also Revs. 2895, 2910, 3388, 3389)

**3071. Hirota, S., and Takada, M., Analysis of non-Newtonian flow by falling-sphere method,** *Bull. Chem. Soc. Japan* **32**, 11, 1191–1194, Nov. 1959.

Authors analyze the steady flow around a sphere falling slowly in a non-Newtonian fluid. They obtain an equation which agrees with their experimental results for aqueous solutions of methyl cellulose and NaCMC, and for nitrocellulose in butyl acetate under conditions displaying only minor departures from Newtonian behavior. The derivation is based on the assumption that the shear stress distribution is the same as for Newtonian fluids and other unjustified assumptions, of which discussion is promised in another paper.

S. Gratch, USA

**3072. Crouch, R. F., and Cameron, A., Graphical integration of the Maxwell fluid equation and its application,** *J. Inst. Petroleum* **46**, 436, 119–124, Apr. 1960.

Maxwell's equation for the friction of a viscoelastic fluid with viscosity varying with pressure is derived. The solution of this equation, giving the friction for the conditions met in gears (a band contact zone), is achieved graphically by the method of isoclines. It is also extended to a circular contact zone (spheres). It is found that the friction deviates from the pure Newtonian value when the relaxation number (relaxation time/one-half of transit time) gets beyond unity. This is applied to some gear scuffing experiments of Borsoff. It is not certain whether they are in the region where viscoelasticity plays a part. Further consideration shows that while it is unlikely that the oil film formation is affected by viscoelasticity, the friction may be.

From authors' summary by K. J. DeJuhasz, USA

**3073. Chebykin, V. A., The experimental determination of the viscosity coefficient in dispersion systems** (in Russian), *Doklady Akad. Nauk AzerbSSR* **14**, 8, 611–616, 1958; *Ref. Zh. Mekh.* no. 9, 1959, Rev. 10440.

The experimental determination of the rheological properties of a clayey dispersion was carried out by the method of capillary viscosimetry on apparatus which consisted of a cistern with an area of free surface equalling 1 $m^2$, a device for recording changes in the level of the liquid in the cistern and two measuring capillaries of the same diameter, $d = 12.6$ mm, and different lengths $L$. By means of tests with a seconds-recording clock, the time required to fill the 2-litre measuring vessel was determined. The author, using Pouiselle's formula, computed the magnitude of the effective kinematic viscosity $\nu$ by excluding the inlet effect. The value $\nu$ calculated in this manner was then used to determine the magnitude of Reynolds number $R$ for each value of the pressure $b$. At the same time a calculation was carried out for the magnitude of the coefficient of resistance

$$\lambda = \frac{2\,gd}{L}\,\frac{b}{v^2},$$

where $v$ is the mean velocity in the capillary, $g$ the acceleration of the forces of gravity. A comparison of the results with the analogous results obtained with a capillary 5.2 cm in diameter made it possible to establish a value for the $R$ number at which the flow of the drilling dispersion proceeds with a constant minimum value

for the viscosity, which corresponded with the maximum possible destruction of the structure.

Note by the abstractor: The statement made by the author that he has established a new effect—the diminution of the limit of fluidity $\tau_s$ in Bingham's equation—is not borne out by the facts, since his own experiments show that the rheological properties of the clayey suspension are not capable of description by means of that equation. R. V. Torner

*Courtesy Referativnyi Zhurnal, USSR*

## Hydraulics

(See also Revs. 2870, 3004, 3027, 3062, 3181, 3199, 3201, 3309, 3323, 3324, 3337, 3341, 3342, 3344, 3361, 3363, 3366, 3367)

**3074. Dzhimsheli, G. A., Determination of normal linear dimensions of the cross section of trapezoidal channels** (in Russian), *Gidrotekh. Stroit.* **30**, 3, 43–44, Mar. 1960.

Author uses the generalized Parlorskii's formula for the Chezy coefficient, writes the equation for the discharge and converts it for the computation of the channel cross section. He recommends computing the cross section dimensions, the depth, if the width at the bottom is given, or this width, if the depth is given, by iteration of the formula for the channel cross section, assuming for the first value of the cross section, required for the given discharge and slope, the triangular or rectangular shape.

Two numerical examples are given, one for the computation of the normal depth and the second one for the width at the bottom. The first iteration gives satisfactory values, therefore a second one is not necessary. The method is applicable also to cross sections with variable slopes and with berms.

Reviewer believes that the iteration may give quicker answer than the trial and error procedure usual for this type of problems. However, if auxiliary graphs are available, like those of Chertousov, the iteration procedure is not necessary for trapezoidal sections. The method remains valid for variable slope and berm sections. Unfortunately, the author does not develop in his paper the extension of the method to variable slope and berm sections for which it, apparently, is much more valuable than for simple sections. B. S. Browzin, USA

**3075. Sasorov, M. P., Formulas for Chézy coefficient** (in Russian), *Gidrotekh. Stroit,* **30**, 2, 53–54, Feb. 1960.

Al'tshul' formula was checked on 8 cross sections of Russian rivers: Angara, Ienisei, Volga, and two mountain rivers. Results disclose large discrepancies, particularly in mountain creeks, up to 169%. Only Ienisei River station gave an excellent agreement within 2%. Absolute hydraulic roughness was computed as extremely variable, between 7 and 140 mm, instead of 20 mm as suggested by Al'tshul'. This unfavorable conclusion is a discussion of a previous paper by V. I. Kalitsun [AMR **13**(1960), Rev. 4637], commending the Al'tshul' formula as the best existing. This discussion is a good example of apparent weaknesses of many empirical formulas. S. Kolupaila, USA

**3076. Stone, J. A., Discharge coefficients and steady-state flow forces for hydraulic poppet valves,** *ASME Trans.* **82** D (*J. Basic Engng.*), 1, 144–154, Mar. 1960.

Experiments were run on a poppet valve operating in hydraulic oil. The experimental values of the flow forces and discharge coefficients were about 25 per cent below the predicted theoretical values. Although there was some scatter in the values of discharge coefficients, there was good correlation for the flow forces.

Flow forces are strongly influenced by the downstream configuration. The smaller the diameter of the downstream chamber the higher the forces.

A poppet configuration was designed and tested which virtually eliminated the flow forces. From author's summary

**3077. Yakhontov, S. A., Evaluation of resistance of a rapid current in a region of relatively small depths** (in Russian), *Gidrotekh. Stroit.* **30**, 6, 40–43, June 1960.

Experiments were carried out with water flow at very small depth, not exceeding 5 times absolute roughness of bottom. For this region the Chézy factor proved to be independant of depth, as was previously demonstrated by H. Morris in the U. S. The roughness coefficient remains constant here, as in the zone of a quadratic law of resistance, which is valid only at high Reynolds numbers. This interesting work has been performed under direction of Professor S. V. Izbash. S. Kolupaila, USA

**3078. Mosonyi, E., The influence of the river lay-out on the navigation** (in Roumanian), *Hidrotebnica* **5**, 9, 297–301, Sept. 1960.

In this study various analytical relationships are established between the factors of the regularization of a river sector, with cascade weirs, as well as their influence on navigation. The savings in navigation time, in working materials, in fuels and oils, resulting from the comparison between the regimen, with and without weirs, are calculated. Thus the developed method allows to calculate the economic efficiency of the river regularizations. These calculations are important especially when a river is utilized in a complex manner, i.e. concerning power economy, navigation, and so on. From author's summary

**3079. Krylov, V. V., Hydraulic design of energy dissipators** (in Russian), *Gidrotekh. Stroit.* **30**, 5, 40–44, May 1960.

An analytical solution is proposed by the author for several basic types of dissipators (as an extension of the method of Obrazovsky, *Gidrotekh. Stroit.* no. 5, 1954) with the aim to determine the reaction $R_x$ exerted by the dissipator and the conjugate depth $b_2$ of the hydraulic jump ($R_x$ and $b_2$ are related through the momentum equation). The basic type, considered at first, is a sill with inclined upstream face on a horizontal bed, the width of the sill at its crest being negligible. Case I, in which the contracted section is located immediately upstream from dissipator, is treated first. The basic assumptions are as follows: The constancy of the depth of the jet immediately upstream from, and on, the dissipator, uniform velocity distribution and hydrostatic pressure distribution in section "A" located at the downstream vertical face of the dissipator (pressure calculated from the free-surface and through the jet to the bottom). In the momentum equation between section "A" and the contracted section, the jet angle with the horizontal at the living point appears as additional unknown, which is determined according to the classical solution of the hydrodynamics for a symmetrically branching jet around a sharp edge. The case in which the contracted section is located a distance upstream from the dissipator is then treated for the same type of dissipator. This case, together with two other dissipator types (in form of teeth, and in two rows) are reduced to case I, on basis of new and not well-founded assumptions. Due to these assumptions, the proposed analytical solutions may hardly be recommended until verified by systematic investigations, delimiting the range of applicability and experimentally verifying either the assumptions or the formula itself. M. M. Boreli, Yugoslavia

**3080. Escande, L., and Dat, J., Remarks on the calculation of surge tanks with spillway** (in French), *C. R. Acad. Sci. Paris* **250**, 8, 1395–1398, Feb. 1960.

**3081. Escande, L., Theoretical method for studying the oscillation in a surge tank with spillway, neglecting the loss of head** (in French), *C. R. Acad. Sci. Paris* **250**, 15, 2651–2654, Apr. 1960.

**3082. Geurst, J. A., Some investigations of a linearized theory for unsteady cavity flows** (in English), *Arch. Rational Mech. Anal.* **5**, 4, 316-346, 1960.

Supplementing his former publications (e.g. AMR **13** (1960), Rev. 5532), author considers a cavitating two-dimensional, unsteady, nonviscous, incompressible flow about a flat plate arranged perpendicularly to the velocity of the uniform flow.

In the first part of his publication the author, using the well-known mathematical model of the re-entrant jet, assumes that the unsteady part of the flow is produced by small oscillations of the flat plate. These time-dependent variations of the flow are thus considered as perturbations of the steady flow. The shape of the cavity, which forms at steady flow behind the flat plate, is determined under the assumption of a constant pressure and constant velocity at the boundary. For the computation of the disturbing effect of the oscillating flat plate, the author makes the assumption that the oscillations are at right angle to the uniform velocity, their amplitude being small compared to the velocity of the flow at the boundary and compared to the width of the flat plate. The additional conditions of the problem are: (1) the unsteady disturbances vanish at infinity; (2) the unsteady disturbance field has neither sources nor vortices at infinity; and (3) the pressure on the boundary of the cavity is constant not only in space but also in time. From the computation it can also be inferred that the oscillations at the rear edge of the cavity are of the same order of magnitude and have approximately the same phase as the oscillations of the flat plate.

The second part of the publication treats the same problem according to the linearized method using both the complex velocity and the complex acceleration potential. The latter method is particularly well suited for the solution of complicated problems. The computations made according to these two methods yield the same results.

Computation of the drag of the flat plate with a cavity concludes this highly interesting and brilliant publication.

M. Strscheletzky, Germany

**3083. Numachi, F., Kawashima, O., and Nakamura, M., Force measurements with cavitation on wing profiles with strong rounded edges, Part 1** (in German), *Sci. Rep. Res. Inst., Tohoku Univ., Japan (B)* **11**, 55-88, 1959/1960.

Water-tunnel test results for seven hydrofoil profiles of 7 to 8% thickness ratio are compared to show changes in cavitation and hydrodynamic force characteristics due to rounding, by circular arcs at leading and trailing edges, of the basic circular sector profile. Motivation is the avoidance of blade edge fragility in very small axial-flow machines. Data are from a series of previous reports in English [AMR **11**(1958), Revs. 4073, 4074; **12**(1959), Rev. 2445; **13**(1960), Rev. 4789]. Reynolds numbers are from 0.3 to 0.9 million; water temperatures and air content are considered (see previous reviews). The emphasis upon simple profile geometry (piecewise constant curvature) seems unnecessary in view of modern blade machining and casting techniques, since well-known airfoil profiles with desirable pressure distribution are available.

The result that the cavitation-free angle-of-attack range (cavitation number $k_d$ up to 2.5) is largest for the sharpest profile suggests, to the reviewer, the effect of suction peaks at the curvature discontinuities near the leading edge. For example, for one of the bluntest profiles (edge radius of 1% chord), the upper and lower surface forward-cavitation zones overlap (for $k_d$ below 0.7), thus indicating that, for the included "smooth entry" angle of attack, the suction peaks are due to thickness distribution. No attention is given to cavitation inception particulars, such as noise versus visual indication, hysteresis and vortex inception [AMR **9**(1956), Rev. 182], but graphs and tables give lift and drag coefficients versus angle of attack and $k_d$.

A. G. Fabula, USA

**3084. Hunter, C., On the collapse of an empty cavity in water,** *J. Fluid Mech.* **8**, 2, 241-263, June 1960.

A very comprehensive treatment of cavitation phenomena, based on the study of spherically symmetric collapse of empty cavities within the flow of water. Although the effects of surface tension and viscosity are neglected, the concept offers a promising agreement with empirical data, thus yielding a complementary analytical tool for practical assessment of cavitation problems in high performance hydraulic machinery. Numerical integration of the principal flow equations is offered for the delineation of pressures, velocities, shock-wave formations and their (often damaging) propagation. The similarity solution equations are used, and the limitations of their validity are described.

In summary, the treatment is indeed a welcome, practical approximation of the cavitation effects, useful to anyone concerned with water (and perhaps similar liquids) cavitation and the consequent energy release in high flow-rate systems, such as turbopumps and similar devices. Altogether, there are fifty (50) equations, the most significant components of which are illustrated in seven (7) figures (graphs).

The research effort is well documented by fourteen (14) references.

C. R. Bell, USA

**3085. Kermeen, R. W., Experimental investigations of three-dimensional effects on cavitating hydrofoils,** Calif. Inst. Technol., Engng. Div. Rep. 47-14 (Contract Nonr-220(12)), 17 pp. + figs., Sept. 1960.

An investigation in the High Speed Water Tunnel of the hydrodynamic characteristics of a family of three-dimensional sharp-edged hydrofoils is described. Four rectangular planform, 6-degree wedge profiles with aspect ratios of 4.0, 2.0, 1.0 and 0.5 were tested over a range of cavitation numbers from noncavitating to fully cavitating flow. The effects of aspect ratio on the flow and cavity configurations and on the lift, drag and pitching moment are discussed. Where data were available the results have been compared with the two-dimensional case.

From author's summary

**3086. Bouligand, G., On the axiom of incompressibility of fluids** (in French), *C. R. Acad. Sci. Paris* **249**, 23, 2452-2454, Dec. 1959.

**3087. Camichel, C., On the vortices in a liquid** (in French), *C. R. Acad. Sci. Paris* **250**, 1, 30-34, Jan. 1960.

## Incompressible Flow

**(See also Revs. 2831, 3071, 3084, 3130, 3131, 3132, 3137, 3139, 3175, 3178, 3194, 3196, 3201, 3202, 3203, 3204, 3329, 3341, 3345, 3356)**

**3088. Mitra, M. K., Resistance on a sphere due to a circular vortex filament in a uniform flow of a perfect liquid** (in English), *ZAMM* **39**, 1/2, 25-30, Jan./Feb. 1959.

This paper gives a formula for resistance (in series of Legendre functions) on a sphere due to a circular vortex filament in a uniform flow of a perfect liquid. The stream function of the motion has been found by Butler's sphere theorem and then the resistance on the sphere is calculated by means of Bernoulli's pressure equation.

From author's summary by E. Storchi, Italy

**3089. Rosenblat, S., Flow between torsionally oscillating disks,** *J. Fluid Mech.* **8**, 3, 388-399, July 1960.

Paper investigates the flows of a viscous fluid contained between two parallel infinite plane disks, which oscillate torsionally

about a common axis. Two cases only are studied: when one disk performs small torsional oscillations and the other is at rest; and when both disks oscillate with the same amplitude and frequency, though with a phase difference of 180°. On the assumption that certain nonlinear inertia terms may be omitted, solutions of the Navier-Stokes equations are developed. The basic parameter is found to be the Reynolds number $nd^2/\nu$, where $n$ is the frequency of torsional oscillations, $d$ the distance between the two disks, and $\nu$ the kinematic viscosity. Solutions for both small and large Reynolds numbers are developed to describe the transverse and radial-axial velocity components. G. Noseda, Italy

**3090. Wooding, R. A., Instability of a viscous liquid of variable density in a vertical Hele-Shaw cell,** *J. Fluid Mech.* **7**, 4, 501-515, Apr. 1960.

Flow of a stratified fluid between two vertical parallel planes is studied for very low Reynolds numbers. Stability of motion is considered with approximate and exact solutions of the equations of motion. Final equations are used to calculate stability criteria for liquid at rest under a vertical density gradient which increases in density with elevation. It is shown that the solution for the critical Rayleigh number obtained by the approximate method is the first term in the corresponding solution by the exact method. Result is consistent with the solution for the velocity distribution which was assumed to be that for Poiseuille flow in the approximate method and closely approaches this form by the exact method.

An asymptotic expansion is obtained for the critical Rayleigh number for a long vertical duct of rectangular section and compared to results of experiments which are briefly described in the paper. Measured critical Rayleigh number is 4% higher than result from exact method and 10% higher than result from approximate method. Author concludes that better agreement with exact solution is significant.

Problem considered is analogous to stability of flow through a porous medium to two-dimensional disturbances. Author shows relationship between physical quantities for approximate method.

W. D. Baines, Canada

**3091. Daily, J. W., and Nece, R. E., Chamber dimension effects on induced flow and frictional resistance of enclosed rotating disks,** *ASME Trans.* **82** D (*J. Basic Engng.*), 1, 217-232, Mar. 1960.

The fundamental fluid mechanics associated with the rotation of a smooth plane disk enclosed within a right-cylindrical chamber have been studied both experimentally and theoretically. In order to acquire further and systematic information pertinent to this problem, which has received much attention in the past, torque data were obtained over a range of disk Reynolds numbers from $10^3$ to $10^7$ for axial clearance-disk radius ratios s/a from 0.0127 to 0.127 for a constant small radial tip clearance, and velocity and pressure data were obtained for laminar and turbulent flows. The existence of four basic flow regimes in the axial gap between the disk and casing wall was verified, and these regimes, the existence and extent of which are governed by the Reynolds number-axial spacing combinations, have been delineated. A new approximate theoretical analysis has accounted for axial-clearance effects for the case of separate boundary layers on the disk and end wall; this theory has been checked against test results. Velocity and pressure data have shown that the concept of a fluid "core" rotation in the case of separate boundary layers must be modified because of secondary flows and skewed boundary layers.

From authors' summary

**3092. Pykhteev, G. N., Cavitation flow of an ideal incompressible fluid in a slot,** *Appl. Math. Mech. (Prikl. Mat. Mekh.)* **24**, 1, 213-220, 1960. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

The generalized problem of plane flow through a Borda orifice is solved by standard mathematical technique of conformal transformations. The jet contracts to a minimum diameter, then expands to fill the whole tube, the free streamline forming a re-entrant jet upstream along the tube walls on second sheet of Riemann surface after Gilbarg and Rock's pattern.

The general case, with non-zero velocity at infinity in the reservoir, is completely and elegantly solved. Particular cases also solved are (1) zero velocity at infinity in the reservoir, (2) the case of no re-entrant jet (conventional Borda jet to infinity), and (3) the combination of (1) and (2).

There is no comparison with experimental results.

Vestiges of the original Russian text unfortunately survive in equations (2.5) but do not obscure the meaning.

A. H. Armstrong, England

**3093. Bourque, C., and Newman, B. G., Reattachment of a two-dimensional, incompressible jet to an adjacent flat plate,** *Aero. Quart.* **11**, 3, 210-232, Aug. 1960.

Study has been made of the reattachment of a two-dimensional, incompressible, turbulent jet to an adjacent, inclined, flat plate.

Two approximate theories, based on Dodds' analysis for the reattachment of a jet to a plate, are developed. The agreement with experiment is fairly satisfactory.

From authors' summary by A. Petroff, USA

**3094. Toba, K., Breslau, M., and Yen, K. T., A study of some fluid mixing problems,** AFOSR TN 60-1023 (Rensselaer Polytech. Inst., Dept. Aero. Engng. TR 6003), 32 pp., Aug. 1960.

An analytical study of some two-dimensional fluid mixing problems is reported. First, the conditions under which similar solutions exist for mixing under streamwise pressure gradients are determined. It is shown that the two streams should have the same Mach numbers, and the main-flow velocity distributions should be of the wedge or exponential flow type.

Next, transverse pressure gradients are also included in the consideration. It is found that the free-stream velocity distributions should also be of the wedge or exponential flow type and, in addition, the total heads of the two fluid streams should be the same.

Finally, the von Kármán-Pohlhausen method is used to allow for more realistic pressure gradients. The dependence of the interface curvature on the "injection velocity," the pressure gradients, etc., are indicated.

From authors' summary

**3095. Wille, R., Karman vortex sheets,** Advances in Applied Mechanics, Vol. 6, New York, Academic Press, Inc., 1960, 273-287.

This comprehensive review article compares theoretical and experimental results on the relative lateral and longitudinal spacing of the vortices behind two-dimensional bodies, as functions of time (distance downstream) and Reynolds numbers. Investigations of the stability of the vortex street are reviewed. Excellent agreement between measured and theoretical velocity distributions within the vortices are show.

A. M. Kuethe, USA

**3096. Jones, J. P., The breakdown of vortices in separated flow,** Univ. Southampton, Dept. Aero. & Astron. Rep. 140, 23 pp., July 1960.

A brief description is given of the phenomenon of breakdown of the vortices above sharp-edged slender wings and it is suggested that it is a form of hydrodynamic instability.

This possibility is investigated theoretically for an infinite vortex core in which both the axial and circumferential velocities are assumed to vary rapidly. The core receives an axisymmetric

(small) disturbance and the conditions are sought for this to grow. Stability is shown to depend upon the existence of radial gradients of the axial velocity. The stability of Hall's vortex is related to the axial pressure gradient and the existence of a characteristic frequency associated with breakdown is predicted. If this can be detected it will hlep to confirm the basic suggestion

Rotating flow through a pipe is shown to be unstable but not for the same reason as a free vortex.

From author's summary

**3097. Power, G., Accelerating body with vortex trail in variable two-dimensional flow** (in English), *ZAMM* **39**, 3/4, 139-146, Mar./Apr. 1959.

Expressions for the resultant hydrodynamic forces on a cylinder of general cross section are deduced when the cylinder is in motion, and when a vortex trail is assumed to form behind. The fluid itself is moving nonuniformly, and its motion is attributed to a number of varying spiral vortices, which can be used as an approximation of any given flow.

From author's summary by E. Storchi, Italy

## Compressible Flow (Continuum and Noncontinuum Flow)

**(See also Revs. 3084, 3130, 3137, 3159, 3165, 3166, 3169, 3197, 3208, 3209, 3214, 3247, 3269, 3278, 3285, 3287, 3302, 3311)**

**3098. Cremer, H., and Kolberg, F., Wind-tunnel corrections for bodies at an angle of attack in a compressible subsonic flow** (in German), *ZAMM* **40**, 1/3, 65-74, Jan./Mar. 1960.

Author considers problem of evaluating velocity corrections for open and closed jet tunnels with circular cross section when test model is inclined to the subsonic flow. Linearized potential equation is used to obtain the velocity potential. Test model is replaced by a source-sink combination (strength equal to free-stream velocity times model volume) located on the body axis. Expressions for the velocity potential are given in terms of integrals containing Bessel functions. Author hopes to report on results of numerical evaluations of the integrals to be carried out on high-speed computer.

W. L. Haberman, USA

**3099. Helliwell, J. B., and Mackie, A. G., The flow past a closed body in a high subsonic stream,** *Quart. J. Mech. Appl. Math.* **12**, 3, 298-313, Aug. 1959.

A solution of Tricomi's equation is obtained for the flow past a thin, doubly symmetric body placed at zero incidence in a high subsonic stream in which sonic velocity is attained along a segment of the body. This flow is the compressible analog of the Riabouchinsky model for incompressible fluids. The singularity in the hodograph plane corresponding to the point at infinity in the physical plane is essentially different from that which occurs in other similar problems. The boundary-value problem is of mixed type and this is shown to lead to a pair of dual integral equations for which the solution is obtained. Numerical results are given which specify the dimensions of the body corresponding to a range of incident Mach numbers. By symmetry the total drag on the body is zero.

From authors' summary by S. Eskinazi, USA

**3100. Fox, R. H., An accurate expression for gas-pressure drop in high-speed subsonic flow with friction and heating,** *ASME Trans.* **82** E (*J. Appl. Mech.*), 4, 747-748 (Brief Notes), Dec. 1960.

**3101. Murphy, C. H., and Piddington, M. J., Aerodynamic properties of ring airfoils in supersonic flight,** *J. Aerospace Sci.* **27**, 12, 954-955 (Readers' Forum), Dec. 1960.

**3102. Woolard, H. W., Tables of properties of some oblique deflagrations in supersonic flow,** Johns Hopkins Univ., Appl. Phys. Lab. Rep. TG-382, 30 pp., Sept. 1960.

Tables of values of the aerothermodynamic parameters for an oblique deflagration in supersonic flow are presented. The deflagration consists of an oblique flame preceded by an oblique shock. The tables either supply directly, or permit the rapid determination of, the heat addition parameter, the Mach numbers in the various regions, and the pressure, temperature, velocity, and density ratios across the shock and flame fronts, for $\gamma = 1.4$, zero flow deflection downstream of the flame front, free-stream Mach numbers of 2, 2.5, 3, 4, 5, 6 and 7, flame-normal Mach numbers of 0.001, 0.002, 0.005, 0.01, 0.02, 0.04, 0.06, and 0.08, and shock-wave flow deflection angles of 0, 2, 4, 6, 8, 12, 16, and 20 degrees.

From author's summary

**3103. Vaglio-Laurin, R., On the PLK method and the supersonic blunt body problem,** AFOSR TN 60-430 (Polyt. Inst. Brooklyn, Dept. Aero. Engng. Appl. Mech., PIBAL Rep. 546), 75 pp., Aug. 1960.

Detailed analysis of the subsonic and transonic portions of the flow field about either very blunt or asymmetric configurations requires successive approximations; these can be carried out in a systematic fashion only if an appropriate convergent perturbation procedure is available. It is shown that, with either "direct" or "inverse" methods of analysis, a straightforward linearization scheme leads to divergent series solutions; however, the situation can be remedied by introducing a simultaneous stretching of coordinates in the spirit of the Poincaré-Lighthill-Kuo (PLK) method. The appropriate perturbations are: (1) For the inverse method coordinates are transformed along body, shock, and intermediate lines so as to annul perturbations of the local resultant velocity; (2) for the integral method the coordinate along the boundary of each strip is shifted so as to annul perturbations of the velocity component that determines the critical point.

The "inverse" method is treated in detail here. The linearization approach is justified by consideration of model transonic flow problems, for which closed form solutions are available. The method is recast so as to permit analysis with initial data prescribed either on the shock (estimated shape) or on the body (estimated pressure distribution). The latter alternative is required when the body profile exhibits a singularity; e.g., a sonic shoulder. Some information about the range of validity for the proposed perturbation procedure is obtained from analysis of the flow about a disk set normal to a low temperature air stream at $M_\infty$ = 4.76; it is found that errors of about 25% in the estimate can be compensated by a first-order perturbation.

Analysis and applications of the linearization approach for the "direct" method are presented in another report.

From author's summary

**3104. Dupuichs, G., Action of a transverse jet in a supersonic flow** (in French), *C. R. Acad. Sci. Paris* **251**, 2, 201-203, July 1960.

**3105. Kogan, A., On supersonic flow past thick airfoils,** *J. Aero/Space Sci.* **27**, 7, 504-508, 516, July 1960.

The inviscid rotational supersonic flow behind the shock wave attached to the sharp leading edge of an airfoil is studied by a transformation of coordinates which introduces the Crocco stream function $\Psi$ as an independent variable. Using expansions in the power series of $\Psi$, an iterative process is developed for the determination of pressure distribution along the airfoil surface.

From author's summary by R. Kawamura, Japan

**3106. Comolet, R., and Sapaly, J., Application of a property of supersonic flow in the conception of a pneumatic pivot** (in French), *C. R. Acad. Sci. Paris* **250**, 23, 3769–3770, June 1960.

**3107. Coupry, G., and Gourbil, L., Vibration of a body of revolution in a slightly supersonic flow** (in French), *Rech. Aéro.* no. 76, 41–47, May/June 1960.

By linearizing and using Fourier series expansions, author proposes a method for calculating aerodynamic characteristics when a slender body of revolution vibrates in a slightly supersonic flow.

Results of experiments made in a transonic wind tunnel are in good agreement with the theory. G. Power, England

**3108. Lilley, G. M., and Spillman, J. J., Ground level disturbance from large aircraft flying at supersonic speeds,** Coll. Aero. Cranfield, Note 103, 20 pp. + figs., May 1960.

The Whitham-Walkden theory for the estimation of the strength of shock waves at ground level from aircraft flying at supersonic speeds is applied to the case of a typical projected supersonic civil transport airplane.

If a figure of 2 lb/sq ft (including a factor of 2 for ground reflection) is taken as an upper limit for the acceptable strength of the bow wave from such an aircraft it is shown that restrictions on the climb and flight plan will be involved. The advantage of the employment of larger engines with or without afterburning is discussed, with reference also to the penalties involved owing to the increase in weight of the aircraft and its direct operating costs.

Finally it is suggested that an aircraft of given volume could be designed, by suitable choice of thickness and lift distribution, to minimize the strength of the shock waves in the far field.

From authors' summary

**3109. Traugott, S. C., Some features of supersonic and hypersonic flow about blunted cones,** Martin Co., Baltimore, Md., Aerophysics Res. Staff RM-64, 54 pp., Mar. 1960.

Author analyzes systematically the pressure distribution along blunted cones with spherical caps by using in parallel the results obtained through Belotserkovskii's integral method as well as experimental results. Shock shapes for various angles of the cone at different Mach numbers are then presented. Finally, the cases in which Belotserkovskii's method cannot be successfully applied are discussed (second shock occurrence or no sonic point on the surface).

Reviewer believes that paper represents valuable material for investigating this field. M. I. Ionescu, Roumania

**3110. Sauer, R., Practical numerical methods of three-dimensional supersonic flow,** AFOSR TR 60-40 (Technische Hochschule, Munchen), 32 pp., Mar. 1960.

The reported research pertains to the following subjects: Linear supersonic flow (l.s.f.) past slender bodies of arbitrary shape; l.s.f. past wing-body combinations; l.s.f. past oscillating bodies; the "area rule" in l.s.f.; two-dimensional unsteady, *non*linear flow; investigations on the numerical methods for linear and *non*linear supersonic flow equations.

From author's summary

**3111. Shmyglevskii, Yu. D., Bodies of rotation having minimal resistance at supersonic velocities,** *Soviet Phys.-Doklady* **4**, 3, 541–543, Dec. 1959. (Translation of *Doklady Akad. Nauk SSSR (N.S.)* **126**, 5, 958–960, May/June 1959 by Amer. Inst. Phys., Inc., New York, N.Y.)

Meridional curve of an axial symmetric body for minimal wave drag is obtained by setting up and solving a variational calculus problem. The solution must represent a physically realistic flow; i.e., satisfy all the equations of continuity, momentum, energy, state and, in addition, the Rankine-Hugoniot relation across the attached shock. If the number of conditions to be satisfied exceeds the number of adjustable parameters, the correct approach is not to relax any of the equations but to introduce more freedom; i.e., allow more adjustable parameters so as to make the problem determinate. Paper deals with precisely such a case, and shows that, to make the problem determinate, a corner or slope discontinuity for the meridional curve must be introduced, the location and magnitude of which is then uniquely determined by the problem. Procedure of numerical computation with method of characteristics is outlined. H. S. Tan, USA

**3112. Lunev, V. V., Flow of a viscous heat-conducting gas at high supersonic speeds about a cone,** *Appl. Math. Mech. (Prikl. Mat. Mekh.)* **23**, 6, 1444–1461, 1959. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

Author attacks the problem of the flow of a viscous, heat-conducting gas about a circular cone at zero angle of attack at high supersonic speeds. The region of the disturbed flow is divided into two subregions: a viscous region in which the flow is considered to be laminar and is described by equations of the boundary layer and a nonviscous region in which the flow is described by equations of an ideal gas. Only a weak interaction between these two regions is considered. Thus the two fundamental solutions of the problem are (1) the solution of the boundary layer on a cone (reduced to a flat plate) and (2) the conical flow (tabulated by Z. Kopal, "Tables of supersonic flow around cones," Cambridge, 1947). In the nonviscous region the author uses the equation for the flow about a circular cone derived by A. Il'iushin ["Law of plane sections," *Prikl. Mat. Mekh.* **19**, no. 6, 1956]. After some operations, the problem in this region is reduced to the problem of Cauchy. In the viscous region the author uses his own equations derived in the conical coordinates ($x$, $y$, $\phi$) in "On the similarity for the flow of a viscous gas at high supersonic speeds about thin bodies," [*Prikl. Mat. Mekh.* **22**, no. 1, 1959]. Modifying Crocco transformation author applies transformation of coordinates ($x$, $y$) into ($\xi = \frac{1}{3}x^3$, $\tau = (\mu/x)\partial u/\partial y$). The system of equations so obtained is solved by means of series expansions.

The calculations are long and for some particular values of parameters are tabulated. It follows that an increase in the boundary-layer thickness for $M_\infty >> 1$ leads to an increase in the frictional resistance and to an increase in the heat flow to the surface of the body. M. Z. v. Krzywoblocki, USA

**3113. Whitcomb, R. T., and Sevier, J. R., Jr., A supersonic area rule and an application to the design of a wing-body combination with high lift-drag ratios,** NASA TR R-72, 14 pp., 1960.

A concept for interrelating the wave drags of wing-body combinations at supersonic speeds with axial developments of cross-sectional area is presented. A swept wing–indented-body combination designed on the basis of this concept to have significantly improved maximum lift-drag ratios over a range of transonic and moderate supersonic speeds is described. Experimental results have been obtained for this configuration at Mach numbers from 0.80 to 2.01. Maximum lift-drag ratios of approximately 14 and 9 were measured at Mach numbers of 1.15 and 1.41, respectively. From authors' summary

**3114. Chernyi, G. G., Effect of slight blunting of leading edge of an immersed body on the flow around it at hypersonic speeds,** NASA TT R-35, 19 pp., June 1960.

Piston combined with blast-wave theory is applied to steady cases of hypersonic flow around slender bodies with blunt leading-edge. Flat plate, cylinder, wedge and cone are considered. Theory of similitude and dimensionality is extensively used and approximate expressions of surface pressure distribution and shape of bow shock wave are obtained. In treatments of the latter two cases author uses an approximation that entire mass of gas in

the perturbed region is concentrated in a zero-thickness layer adjacent to the shock wave. Possibility of reducing total drag of a cone by blunting of the nose is pointed out. Results are compared with those of more exact theories and experiments for several of the cases are considered. R. Kawamura, Japan

**3115. Legendre, R., Hydraulic analogy for the study of hypersonic flow** (in French), *C. R. Acad. Sci. Paris* **250**, 23, 3771–3772, June 1960.

**3116. Tsynkova, O. E., Motion of a gas in channels of finite length with variable counter pressure** (in Russian), *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk* no. 3, 15–24, May/June 1959.

It has been known for some time that, for flow in an infinite channel with permeable walls, the presence of a slit in the walls in a plane perpendicular to the axis of the channel has a stabilizing effect on the position of the compression shock. The present paper examines, in the linearized approximation, the case of flow in a finite channel with two such slits at one of which there is suction and at the other injection, and with a variable closing aperture at the end of the channel. The solution is found in two cases: (1) basic flow in a channel divided by compression shock into super- and subsonic flows; (2) the basic flow in the channel is purely subsonic.

L. M. Milne-Thomson, USA

**3117. Koga, T., The structure of strong shock waves of stable monatomic molecules,** AFOSR TN 60-1344 (Univ. So. Calif. Engng. Cntr. Rep. 83-201), 59 pp., Oct. 1960.

The present theory of plane shock waves of a monatomic gas is based on the following three hypothetical assumptions:

(1) The distribution function $f(c_x, c_y, c_z)$ is separable and $f = f_x(c_x) f_{yz}(c_y, c_z)$, where the main flow is in the x-direction.
(2) The characteristics of $f_x$ are evaluated by the degree of unsymmetry.
(3) For the collision term of the Boltzmann equation, an approximation is taken similar to what was proposed by Bhatnagar and others. It is proposed that the persistence of state of a particle after its collision is to be taken into account.

According to assumptions (1) and (2), two series of distribution functions are designed. There is no serious mathematical difficulty in solving five equations of moments by which the five unknown variables involved in an assumed distribution function are determined. The results are compared with those of other authors. According to the results, initial assumptions (1) and (2) seem to be plausible as long as theory is confined in a limited domain of interest so that moments of higher orders with respect to $c_y$ and $c_z$ are not involved in the theory. From author's summary

**3118. Raymond, J. L., Piston theory applied to strong shocks and unsteady flow,** *J. Fluid Mech.* **8**, 4, 509–513, Aug. 1960.

The application of piston theory to both steady and vibratory two-dimensional airfoil theory [see AMR **10**(1957), Revs. 2630, 2631; **12**(1959), Rev. 478] is extended to the case where the leading-edge shock is strong and the flow therefore nonisentropic. Formulas relating local pressure coefficient to piston velocity are presented for both isentropic and strong shock flow.

Force and moment coefficients for a particular worked example are found to agree with results of shock-expansion theory to within a few per cent. A. H. Armstrong, England

**3119. Nagamatsu, H. T., Workman, J. B., and Sheer, R. E., Jr., Oblique shock relations for air at Mach 7.8 and 7200 R stagnation temperature,** *ARS J.* **30**, 7, 619–623, July 1960.

Oblique shock relations for dissociated air in thermodynamic equilibrium have been calculated for a free-stream flow Mach number of 7.8 in the test section with an equilibrium stagnation temperature of 7200 R in the reservoir. The results for the density, pressure and temperature ratios, and flow deflection across the shock wave are presented as functions of the shock wave angle. Complete equilibrium has been assumed in the calculations, utilizing the best available thermodynamic properties of air for the region considered. The real gas oblique shock relations have been experimentally verified by testing an adjustable two-dimensional wedge model in a hypersonic shock tunnel. Correlations of the calculated and experimental pressure ratios and shock-wave angles are presented as functions of flow deflection angle. When allowance was made for the boundary-layer displacement effect, the correlation was seen to be good. The reservoir conditions were selected to insure that the flow in the shock tunnel would be close to equilibrium. From author's summary

**3120. Blanco, E., Cabannes, H., and Kuntzmann, J., Curvature of attached shock waves in steady axially symmetric flow,** *J. Fluid Mech.* **7**, 4, 610–616, Apr. 1960.

Numerical values of the ratio between the initial radii of curvature of the attached shock wave and of the cone are given. The free-stream Mach number ranges from 1 to around 20 for body angles from 5° to 25° in intervals of 2.5° and from 30° to 55° in intervals of 5°. For small body angles, the asymptotic formula as given by P. S. Rao, *Aero. Quart.* **7**, p. 135 (1956), is applicable.
C. B. Ludwig, USA

**3121. Gerber, N., and Bartos, Joan M., Calculation of flow-variable gradients behind curved shock waves,** *J. Aerospace Sci.* **27**, 12, 958–959 (Readers' Forum), Dec. 1960.

**3122. Cabannes, H., On the attachment of shock waves in two-dimensional flow** (in French), *C. R. Acad. Sci. Paris* **250**, 11, 1968–1970, Mar. 1960.

**3123. Appleton, J. P., The structure of a centred rarefaction wave in an ideal dissociating gas,** Univ. Southampton, Dept. Aero. & Astron. Rep. 136, 43 pp. + figs., Apr. 1960.

The effect of finite rates of molecular dissociation and atomic recombination on the structure of a centered rarefaction wave in an ideal dissociating gas is studied theoretically. By using the method of characteristics, the equations describing the flow are solved numerically for one particular set of initial conditions. From the results of the theoretical analysis and the numerical computations, the possibility of carrying out experimental work on this flow configuration is discussed.

From author's summary

**3124. Ogawa, A., High speed flow in a Laval nozzle,** *Trans. Japan Soc. Aero. Space Sci.* **2**, 3, 77–82, 1959.

In the present paper author discusses a nozzle flow given by an exact solution of the Tomotika-Tamada equation in the hodograph method. Several streamlines in the subsonic and transonic region in the physical flow are computed, each of which may be taken as the wall of the nozzle in the nozzle design. Flow conditions on the sonic line as well as on the limiting Mach wave are also calculated. These may serve as the starting data for the continuation to the purely supersonic flow by the method of characteristics. Further, occurrence of limit lines is discussed in an analytic solution for the nozzle flow. From authors' summary

**3125. Weber, H. E., Ejector-nozzle flow and thrust,** *ASME Trans.* **82** D (*J. Basic Engng.*), 1, 120–130, Mar. 1960.

The diverging shroud ejector nozzle (for aircraft propulsion) is analyzed using one-dimensional theory. Author assumes interface

between streams to have a prescribed axial pressure variation and that both streams retain isentropic cores at exit. Thrust coefficient and flow rate equations are developed. Test data are shown to be in good agreement with author's theoretical predictions.
R. H. Page, USA

**3126. Clarke, J. F., On the sudden contact between a hot gas and a cold solid,** Coll. Aero., Cranfield, Rep. 124, 34 pp., Jan. 1960.

Solutions are presented of the linearized small perturbation equations of one-dimensional unsteady motion in a viscous, heat-conducting, thermally and calorically perfect continuous gas of semi-infinite extent. The perturbations about an initially uniform motionless state in the gas are caused by application of a semi-infinite heat-conducting solid at the free boundary of the gas, the solid being initially isothermal and at a temperature different from that of the gas. Ensuing temperature, pressure and velocity histories of wave like nature are given for Prandtl numbers of 3/4 and zero, values which simplify the solutions. For times which are large relative to the molecular collision times, the velocity and pressure disturbances in the gas are found to be concentrated around a wave front moving at sound speed, while near the interface, classical pure-conduction temperatures are displayed. Related work cited is that of Cole and Wu [Cole, J. D., and Wu, T. Y., *J. Appl. Mech.* **19**, p. 209, 1952] who neglected viscosity in a comparable analysis, Lagerstrom, Cole and Trilling [Lagerstrom, P. A., Cole, J. D., Trilling, L., "Problems in the theory of compressible fluids," GALCIT Report N6-ONR, 1949] who considered viscous waves in non-conducting media, and Hamin [Hanin, M., *J. Math. Phys.* **36**, 3, p. 234, 1957]. Reviewer notes that subsequent to the subject paper, work by Moore and Curtis appeared [Moore, F. K., and Curtis, 1960 Heat Transfer & Fluid Mechanics Institute, Stanford University Press, 1960] dealing with the temperature and chemical composition histories due to a thermal disturbance at the boundary of a dissociating binary gas mixture, taking into account rate processes but neglecting velocity and pressure fluctuations. Subject paper casts light on the role of the coupled effect of the velocity and pressure fluctuations on the thermal field.
M. H. Bloom, USA

**3127. Mackie, A. G., Singularities in the hodograph plane arising from problems of flow past bodies,** Brown Univ., Div. Appl. Math. TR 36 (Contract Nonr 562 (07)), 31 pp. + figs., Aug. 1960.

An investigation is carried out of the singularities which occur in the hodograph plane at the point corresponding to the point at infinity in the physical plane when a body is placed in an infinite two-dimensional stream. The primary aim is to identify these singularities for compressible fluids, as the hodograph method offers one of the best methods of attack in the high subsonic and transonic regions. However, as a first step, an examination is made of singularities in incompressible flow in which classes of exact solutions are known in both physical and hodograph planes. Analogous singularities are then identified for compressible flow. Considerable progress can be made in certain cases for gases with a general equation of state and this leads to the determination of a class of flows with free streamlines for this general fluid. In other cases the fundamental singularity can only be determined explicitly for a flow governed by the Tricomi equation for thin bodies in a near sonic stream.
From author's summary

**3128. Sims, J. L., On conical-flow upwash,** *J. Aerospace Sci.* **27**, 12, 952-953 (Readers' Forum), Dec. 1960.

# Boundary Layer

(See also Revs. 3089, 3093, 3112, 3159, 3190, 3202, 3214, 3245, 3248, 3337, 3345)

**3129. Yasuhara, M., Experiments on axisymmetric boundary layers along a long cylinder in incompressible flow,** *Trans. Japan Soc. Aero. Space Sci.* **2**, 3, 72-76, 1959.

Experiments on axisymmetric boundary layers along a long cylinder were made especially to investigate the effect of transverse curvature on the velocity profile. Laminar velocity profiles were measured and compared with theoretical ones with good accuracy. A representative profile was plotted to see the effect of transverse curvature, which showed small but obvious effect accompanied by increasing skin friction.

The transition of the flow from laminar to turbulent was observed, and its Reynolds number was estimated to occur at 1.2-1.8 × 10 in the present experiment. The turbulent profile was also measured and plotted by using the coordinates to express the wall law deduced by Richmond, from which it was estimated that, as the ratio of the momentum thickness to body radius increases, the profile near the outer layer tends to bend down relative to the line of logarithmic wall law.
From author's summary by P. A. Libby, USA

**3130. Ando, S., A calculation of distributed suction required to control laminar boundary layer,** *Trans. Japan Soc. Aero. Space Sci.* **2**, 3, 64-71, 1959.

An integral method due to I. Tani is extended to the case of a laminar boundary layer with arbitrary pressure distribution and suction at the wall. Both incompressible and compressible flows are considered. Particular emphasis is given to the determination of the distribution of suction necessary to eliminate separation with an adverse pressure gradient.
P. A. Libby, USA

**3131. Napolitano, L. G., and Ferri, A., On a new approach for the solution of non-similar boundary layer problems,** AFOSR TN 59-356 (Polyt. Inst. Brooklyn, Dept. Aero. Engng. Appl. Mech., PIBAL Rep. no. 507), 33 pp., Mar. 1960.

The nonsimilar solution of Prandtl boundary-layer equations for the isobaric flow field of an incompressible viscous fluid is considered. It is shown that the basic equations can always be reduced to a total differential equation depending on a single parameter $\beta$. It is proposed that the flow field can be described by means of "inner" and "outer" profiles joined along a continuous matching line lying within the field itself. The inner and outer profiles are single parameter profiles derived from the solution of the total differential equation upon which not all the pertinent physical conditions are necessarily imposed. It is also proved that when the similarity conditions in the boundary conditions are not imposed simultaneously at the two points (zero and infinity), six classes of single parameter profiles can be generated corresponding to the different types of homogeneous boundary conditions at zero and of boundness at infinity. Each class contains infinite profiles for the infinite values of the parameter $\beta$. The procedure for matching the profiles has been derived and discussed in detail and a practical case of application is presented.
From authors' summary by N. Z. Azer, Egypt

**3132. Cheng, S.-I., and Levy, R. H., The boundary layer in a corner,** AFOSR TN 59-1165 (Princeton Univ., Dept. Aero. Engng. Rep. 485), 45 pp., Nov. 1959.

The incompressible boundary layer between two flat plates intersecting at an arbitrary angle is treated. The intersection is parallel to the incoming flow, and the leading edges are perpendicular to it. The flow in the corner is obtained in series of powers of $\rho$, dimensionless distance from the corner. The leading

power of $\rho$ is a function of plate angle, determined by taking creeping flow as the lowest approximation. Higher terms are obtained by iteration, with additional modes of creeping flow being introduced whenever appropriate. An asymptotic solution for $\rho$ large is matched numerically to the series solution. Results for a wide range of plate angles are given in graphical and tabular form. A. C. Pipkin, USA

**3133. Turcotte, D. L., A sublayer theory for fluid injection into the incompressible turbulent boundary layer,** *J. Aerospace Sci.* **27**, 9, 675–678, 711, Sept. 1960.

In sublayer, eddy viscosity is proportional to $\sinh^2 (by_+/13.89)$, where $y_+$ is nondimensional distance from wall and $b$ is free parameter. As remarked by author, eddy viscosity should grow as $y_+^3$ or $y_+^4$ near wall rather than as $y_+^2$, the variation for $\sinh^2 (by_+/13.89)$. Velocity in stream direction is assumed to depend only on distance from wall. Shear stress in sublayer is found to increase with distance from wall. It is assumed that shear stress at outer edge of sublayer is unaffected by fluid injection. This value of shear stress is then the wall stress at same station of turbulent boundary layer without injection. The decrease in wall shear stress caused by injection follows. Value to be given to free parameter $b$ is discussed and ratio of shear stress with injection to that without injection is given for three reasonable values of $b$. For small value of injection parameter, reduction in shear stress is independent of $b$. Predicted reduction in wall shear stress with injection agrees well with experimental reduction for one choice of $b$. Experimental velocity profiles are given to show that injection does not noticeably affect turbulent boundary layer outside sublayer. N. Tetervin, USA

**3134. Karhan, K., The axial frictional resistance of long cylinders in turbulent flow,** *J. Ship Res.* **3**, 2, 24–28 (Technical Note), Oct. 1959.

In ship model extrapolation, the frictional resistance of the ship and model surfaces is generally taken to be equal to that of a flat plate having the same length and area. Using the power law for the velocity distribution in the boundary layer, author has derived correction factors to the specific friction coefficient for flat plates as functions of transverse curvature and Reynolds number. Results shown indicate that corrections for models will be greater than those for ships, thus explaining one of the reasons for scale effects. Continuation of author's work should prove of great importance in the theory of model testing.

R. B. Couch, USA

**3135. Elder, J. W., An experimental investigation of turbulent spots and breakdown to turbulence,** *J. Fluid Mech.* **9**, 2, 235–246, Oct. 1960.

Experiments were made on a flat plate in zero pressure gradient, mainly in a wind tunnel but with a few supporting experiments in a water channel, using dye for visualization. Results confirm assumptions made by Emmons in AMR 5(1952), Rev. 486. In particular it is shown that turbulent region is simply the sum of the turbulent areas that would occur if all spots grew independently. Experiments with sparks to disturb boundary layer showed that turbulent spot is initiated if maximum velocity fluctuation exceeds a certain critical value. This critical value is about 1/5 of freestream velocity and appears to be independent of Reynolds number and of spatial extent of disturbance. In particular, turbulent spot can be initiated by suitable disturbance, even at Reynolds numbers so low that *small* disturbances are damped out. Results are compared with those of Klebanoff and Tidstrom [AMR **13**(1960), Rev. 4094]; author concludes that slight differences are not significant. W. A. Mair, England

**3136. Elder, J. W., The flow past a flat plate of finite width,** *J. Fluid Mech.* **9**, 1, 133–153, Sept. 1960.

An experimental study of the incompressible laminar and turbulent boundary layer near the edge of a finite flat plate is reported for the Reynolds number range $10^4$ to $10^6$. The laminar velocity field does not contain any secondary flow. The excess skin friction due to the small local radius of curvature at the edge agrees well with theories by Glauert & Lighthill and by Varley. Turbulence wedge originates from a nearly point-like region on the edge of the plate, well upstream of the normal transition zone in the middle. Within this wedge-shaped turbulent region a weak secondary flow begins to develop, driven by the anisotropic Reynolds stresses, direction of rotation being toward the edge near the plate surface (Prandtl's secondary flow of the second kind). Although the secondary flow is localized within a few boundary-layer thicknesses, it results in a constant increase of the total friction-drag coefficient by an amount 0.0004, independent of the Reynolds number and the plate width. This result brings various formulations of the drag coefficient of a finite plate within ±1% discrepancies from Schoenherr's formula.

The reviewer finds this paper quite informative and useful for practical applications as well. F. R. Hama, USA

**3137. Dem'ianov, Iu. A., Mixing of boundary layers at a liquid or gas interface,** *Appl. Math. Mech.* (*Prikl. Mat. Mekh.*) **23**, 2, 520–527, 1959. (Pergamon Press, Inc., 122 E. 55th St., New York 22, N. Y.)

A semi-infinite plate at time $t = 0$ has fluid flowing past it with viscosity coefficient $\mu_1$ and kinematic viscosity $\nu_1$. Beginning at this moment, another fluid with properties $\mu_2$ and $\nu_2$ flows past the plate. The speed, $V_\infty$, of the fluid is assumed constant. The problem is to determine the mixing process of the boundary layers.

Analysis is first carried out for incompressible flow. Solution depends on only two dimensionless independent variables: $\eta = y/\sqrt{x\nu_1/V_\infty}$, $\xi = x/V_\infty t$. By means of an integral method based on third-degree velocity profiles, partial differential equations for the boundary layers are converted into two first-order ordinary differential equations, with boundary-layer thickness and the equation of the surface dividing the fluids as the unknowns. Appropriate conditions at the wall, outer edge of the boundary layers, and at the dividing surface must of course be satisfied. An approximate solution, based on a power series method, is obtained.

Author shows that the analysis given can be extended to the case of compressible flow past a plate, with fluids such that $\mu_1\rho_1 = \text{const}$ and $\mu_2\rho_2 = \text{const}$.

M. Morduchow, USA

**3138. Czarnecki, K. R., and Sevier, J. R., Jr., Investigation of effects of roughness, surface cooling, and shock impingement on boundary-layer transition on a two-dimensional wing,** NASA TN D-417, 41 pp., June 1960.

An investigation was conducted to determine the effects of single-element surface roughness, surface cooling, and shock impingement on boundary-layer transition on a two-dimensional wing. Tests were conducted at Mach numbers of 1.61 and 2.01, and over a Reynolds number per foot range from $0.5 \times 10^6$ to $9.5 \times 10^6$, approximately.

Transition at zero heat transfer was apparently strongly influenced by surface conditions. Heating or cooling the model surface had little effect on transition when small and nearly undetectable surface roughness existed, but with surface roughness eliminated, surface cooling was quite effective in increasing transition Reynolds number. Installation of a sharp-edged plate or wedge so that the flat side of the wedge was alined with the stream had no effect on transition. Deflection of this surface in

either the positive or negative direction generally resulted in large reductions in the values of transition Reynolds number.

From authors' summary by M. S. Macovsky, USA

**3139. Smith, A. M. O., Remarks on transition in a round tube,** *J. Fluid Mech.* **7,** 4, 565–576, Apr. 1960.

This article has a twofold purpose: (1) To analyze the available theoretical and experimental knowledge concerning flow in the inlet region of a smooth round tube, and (2) to point out that the $e^9$ amplification factor method of Smith and Gamberoni apparently predicts natural transition correctly over a significant fraction of the entire inlet length of the tube. The successful prediction indicates, but does not prove, that flow in a smooth round tube becomes turbulent at higher Reynolds numbers because transition occurs in the inlet length, not in the fully developed Poiseuille regime.

From author's summary by J. V. Becker, USA

**3140. Rogers, E. W. E., and Hall, I. M., An investigation at transonic speeds of the performance of various distributed roughness bands used to cause boundary-layer transition near the leading edge of a cropped delta half-wing,** Aero. Res. Counc. Lond. Curr. Pap. 481, 46 pp., 1960.

Distributed roughness bands of no. 320 and no. 500 carborundum were found to be effective in causing boundary-layer transition if they extended over the first 5% and 10%, respectively, of the local chord. Use of larger grain sizes, or increases in the band width for a given grain size resulted in a drag penalty.... The wing lift and pitching moment were only slightly modified by the presence of any of the roughness bands tested, but this result would not of course necessarily apply to wings of other planforms or section shapes. The test Reynolds number was about 2.7 millions.

From authors' summary by D. G. Hurley, Australia

**3141. Mandl, P., Effect of standing vortex on flow about suction aerofoils with split flaps,** Nat. Res. Counc., Canada, Aero Rep. LR-239, 33 pp., Jan. 1959.

This investigation extends the theory of flow about suction airfoils with split flaps given in a previous paper [AMR **12**(1959), Rev. 5670]. In the earlier paper in the calculation of the flow past a two-dimensional airfoil with a split flap with a suction slot on the upper surface of the slap the Kutta-condition could be fulfilled at the tip of the flap, not at the trailing edge of the airfoil. In this extension of the theory a standing vortex with the angle between airfoil and flap is taken into account. The strength and the position of the vortex are calculated by applying Kutta condition both at the tip of the flap and at the trailing edge of the airfoil. The standing vortex increases the lift coefficient thus widening the discrepancy between theory and experiment reported in the first paper. This discrepancy can be explained only by viscous effects.

K. Gersten, Germany

**3142. Libby, P. A., The homogeneous boundary layer at an axisymmetric stagnation point with large rates of injection,** WADD TR 60-435 (Polyt. Inst. Brooklyn, Dept. Aero. Engng. Appl. Mech., PIBAL Rep. 605), 130 pp., Aug. 1960.

This report presents a theoretical analysis of the boundary layer at an axisymmetric stagnation point with large rates of air injection. The results of a previous investigation indicated that, for localized mass transfer in the stagnation region, the rates of injection are considerably greater than those usually treated. The exact stagnation point boundary-layer equation is integrated numerically for an approximate representation of the gas properties. The two point boundary conditions are treated in a new manner which is useful for various boundary-layer and mixing problems. The exact solutions indicate that for large rates of injection the boundary layer is closely represented by an inner isothermal shear flow and by an exterior, relatively thin region, in which the flow variables change to their free-stream values. An integral method based on profiles suggested by the exact solutions is developed and shown to lead to accurate predictions of the integral thicknesses which are of interest for a study of the down stream influence of the stagnation point mass transfer.

From author's summary

**3143. Lam, S. H., and Rott, N., Theory of linearized time-dependent boundary layers,** AFOSR TN-60-1100 (Cornell Univ., Graduate School of Aero. Engng.), 42 pp. + figs., July 1960.

The problem of a laminar two-dimensional time-dependent boundary layer whose free-stream velocity is given by $U_\infty(x,t) = U_0(x) + \varepsilon U_1(x) e^{\omega t}$ (with $\varepsilon << 1$ and $U_0(x)$ and $U_1(x)$ of the same order of magnitude) has found some interest in the past. An approximate solution of the problem has been given by Lighthill (1954). Obviously unknown to the present authors, an exact solution for the flat-plate problem (also for temperature boundary layers), using series expansions, has been developed by S. Gibellato [*Atti Accad. Sci. Torino Cl. Sci. Fis. Mat. Natur.* **89,** 180–192, 1955 and **91,** 152–170, 1957, in Italian].

The major goal of the present investigation is to resolve certain mathematical peculiarities of this category of unsteady, flat-plate flows, but most of the analysis is not restricted to the flat plate. The question of the joining of the low-frequency and high-frequency series solutions is given special attention. It is shown that when $U_0(x)$ is proportional to a power of $x$ there is a unique solution, that the low-frequency series solution generally has an infinite radius of convergence, and that consequently any high-frequency series must be an asymptotic expansion. High-frequency eigen solutions provide the freedom necessary for the joining. Extensive numerical computations, performed to substantiate the theory, are also presented.

W. Wuest, Germany

**3144. Chung, P. M., and Anderson, A. D., Heat transfer to surfaces of finite catalytic activity in frozen dissociated hypersonic flow,** NASA TN D-350, 36 pp., Jan. 1961.

Paper considers boundary layer over catalytic surfaces (first-order reaction) in a binary mixture. Gas phase reactions are frozen and transpiration through the wall is included. Basic equations are put into forms similar to those met with in incompressible boundary-layer theory and, in dealing with flat plate and highly cooled blunt body problems, pressure gradient effects are neglected. With suitable assumptions about $\rho\mu$ (density times viscosity) variations, authors claim decoupling of momentum and species equations from energy equation. Reviewer comments that Eq. [4] in paper should contain a term

$$-\mu\left[\frac{1}{\mathrm{Pr}}-\frac{1}{\mathrm{Sc}}\;\frac{\partial c_1}{\partial y}\right]\int_0^T (C_{p_1}-C_{p_2})dT$$

on the right hand side for a binary mixture (in notation of paper). Term can only be neglected if either $Pr = Sc$ and/or $C_{p_1} = C_{p_2}$, but authors do not state this fact. Calculations are made later for $Sc = 0.72$, so possibly $Pr = Sc$ is implied here. Unless above conditions are satisfied energy equation does not decouple. Also, reviewer infers from Eq. [27] that it must be molecular species which transpires through wall, although this is not explicitly stated anywhere.

Integral methods, with fourth and fifth-degree polynomials for transformed velocity, and concentrations profiles, are used to reduce equations to ordinary differential equations, and similarity transpiration and local similarity concepts permit further simplification. Examples of solutions are given for a flat plate and cone with spherical nose cap, the latter being valid for practical hypersonic bodies at altitudes of between 150,000 and 250,000 ft. Com-

parisons with exact solutions indicate that local similarity solutions are quite accurate for bodies whose equilibrium heat transfer and surface pressure distributions are similar.

J. F. Clarke, England

## Turbulence

(See also Revs. 3089, 3093, 3129, 3133, 3134, 3135, 3136, 3139, 3241, 3306, 3310, 3350, 3352)

**3145. Munk, M. M., Elementary mechanics of turbulent fluid motion,** AFOSR TR 60-6 (Catholic Univ. Amer., Dept. Mech. Aero. Engng.), 146 pp., Mar. 1960.

A novel and extremely interesting way of describing turbulence by introducing a unit of fluid, the "lump." Proper definition of the lump permits the reduction of statistics to a minimum. Also, many relationships are derived on that basis without the help of empirical methods, or at least without quantitative empirical information. The text may be difficult to understand for the beginner, but is very informative to anyone familiar with more conventional methods of describing turbulence.

H. A. Einstein, USA

**3146. Shimanuki, A., On the probability distribution of the autocorrelation function in atmospheric turbulence,** *Sci. Rep. Tohoku Univ., Japan, Ser. 5, Geophysics* **11**, 2, 132-147, Sept. 1959.

The fluctuations of one velocity component can be represented by the stochastic relation of the velocity at any two times. This yields the joint-probability density function $P(u_{t1}, u_{t2})$. By making the assumption that curves of constant $P$, for a given value of $|t_1 - t_2|$ are similar ellipses—which seems plausible and should be tested—it is possible to derive the statistical characteristics of the velocity component. In particular such effort has gone into the derivation of the probability distribution of the auto-correlation function of the velocity component.

J. A. Businger, USA

**3147. Howells, I. D., An approximate equation for the spectrum of a conserved scalar quantity in a turbulent fluid,** *J. Fluid Mech.* **9**, 1, 104-106, Sept. 1960.

In two previous papers [AMR **12**(1959), Revs. 5672, 5673] an analysis was outlined of the small-scale variations of a quantity like temperature in a turbulent field. Present paper extends analysis for fluids of large conductivity to region of smaller wave numbers. Interaction of velocity and temperature fields is represented by a combination of eddy conductivity due to smaller eddies and a straining action due to larger eddies. Resulting approximate equation for spectrum should be valid for equilibrium range of wave numbers at least. Author suggests that similar approach should be instructive for kinetic energy spectrum.

W. D. Baines, Canada

**3148. Pantchev, S., On the statistical theory of turbulence** (in French), *C. R. Acad. Sci. Paris* **250**, 4, 661-662, Jan. 1960.

**3149. Matschinsky, M., Mean value of tensor and the theoretical description of the phenomenon of turbulence** (in French), *C. R. Acad. Sci. Paris* **249**, 23, 2480-2482, Dec. 1959.

**3150. Bouligand, G., Universal theory of turbulent movement** (in French), *C. R. Acad. Sci. Paris* **250**, 11, 1948-1950, Mar. 1960.

**3151. Comte-Bellot, Genevieve, Coefficient of dissymmetry and of flattening of the derivatives in time, on the longitudinal speed fluctuations of liquid flow in the vicinity of a border** (in French), *C. R. Acad. Sci. Paris* **249**, 23, 2483-2485, Dec. 1959.

**3152. Ling, S. C., Heat-transfer characteristics of hot-film sensing element used in flow measurement,** *ASME Trans.* **82** D (*J. Basic Engng.*), 3, 629-634, Sept. 1960.

This describes an important new development in turbulence measuring apparatus, particularly for liquid flows. Used in a circuit similar to the constant-temperature (negative feedback) type of hot-wire anemometer, a platinum film coated on glass has already yielded new basic turbulence data. A semi-theoretical description of the dynamic heat transfer is rather sketchy, but reviewer has learned from users that satisfactory empirical calibrations are obtained.

S. Corrsin, USA

**3153. Gregory, N., Transition and the spread of turbulence on a 60° swept-back wing,** *J. Roy. Aero. Soc.* **64**, 597, 562-564 (Tech. Notes), Sept. 1960.

## Aerodynamics

(See also Revs. 3093, 3098, 3099, 3101, 3105, 3107, 3112, 3114, 3118, 3120, 3129, 3133, 3141, 3153, 3208, 3211, 3248, 3278, 3287, 3301)

**Book—3154. Carroll, R. L., The aerodynamics of powered flight,** New York, John Wiley & Sons, Inc., 1960, ix + 275 pp. $8.50.

Book is intended to give an introduction to aeronautical engineering at the undergraduate level.

The 16 chapters contain mechanics of flight, incompressible and compressible aerodynamics, propeller analysis, airplane performance and stability, and test facilities. Each chapter is followed by a number of problems, with answer to some of them.

Over-all impression is that the author in many cases uses too much elementary mathematics and too little physical explanation and technical discussion. Against the representation of the aerodynamic fundamentals some objections may be raised.

E. Petersohn, Sweden

**3155. Whitcomb, C. F., Critzos, C. C., and Brown, Philippa F., An investigation of wing and aileron loads due to deflected inboard and outboard ailerons on a 4-percent-thick 30° sweptback wing at transonic speeds,** NASA TN D-620, 282 pp., Jan. 1961.

The model was a sting-mounted wing-body combination, and pressure measurements over one wing panel and one inboard and two outboard ailerons were obtained for angles of attack from 0° to 20° at deflections up to ±15°. Reynolds number was about $7.4 \times 10^6$ based on the wing mean aerodynamic chord. Positive deflections of the ailerons caused significant added loadings over those wing sections not spanned by the controls. The loading shapes over the ailerons are irregular and, therefore, would be difficult to predict by theory in the transonic speed range.

From authors' summary

**3156. Elle, B. J., On the breakdown at high incidences of the leading edge vortices on delta wings,** *J. Roy. Aero. Soc.* **64**, 596, 491-493 (Tech. Notes), Aug. 1960.

**3157. Garner, H. C., Charts for low-speed characteristics of two-dimensional trailing-edge flaps,** Aero. Res. Counc. Lond. Rep. Mem. 3174, 35 pp., 1960.

**3158. Spence, D. A., and Beasley, J. A., The calculation of lift slopes, allowing for boundary layer, with applications to the RAE 101 and 104 aerofoils,** Aero. Res. Counc. Lond. Rep. Mem. 3137, 19 pp., 1960.

**3159. Arabian, D. D., Investigation at transonic speeds of loading over a 30° sweptback wing of aspect ratio 3, taper ratio 0.2, and NACA 65A004 airfoil section mounted on a body,** NASA TN D-421, 79 pp., June 1960.

Aerodynamic loads are presented for a wing-body combination at Mach numbers from 0.80 to 1.03 at angles of attack up to 26°. Two wings, both with 30° of quarter-chord sweep, aspect ratio 3, and 4-percent-chord thickness but of different construction, were tested. One wing was constructed of solid steel, the other of plastic with a steel core. The twist distributions for the wings were calculated from experimental influence coefficients and pressure data. Studies of flow in the boundary layer are presented.

From author's summary by H. Bergh, Holland

**3160. Ostoslavskii, I. V., and Grumondz, T. A., On the relation between the generation of a lift force on a wing and the character of the flow in the boundary layer,** NASA TT F-26, 12 pp., May 1960.

Paper presents the results of a theoretical and experimental investigation of the mechanism involved in the generation of lift on a wing as the wing is started from rest. It is shown theoretically that as the wing motion is started, vortices form near the wing leading edge on both surfaces due to viscous effects, and these vortices then flow downstream. At angle of attack the vortices on the lower surface reach the trailing edge first, leaving a surplus of vortices on the upper surface after steady motion is reached which are related to the general circulation or lift. A comparison of experiment with theory is included.

From authors' summary

**3161. Warner, R. W., and Packard, Barbara B., A method for calculating the aerodynamic forces due to arbitrary, time-dependent downwash for a class of thin, flexible wings at supersonic speeds,** NASA TN D-142, 64 pp., Feb. 1960.

Indicial aerodynamic influence coefficients are determined for a series of thin flexible wings with supersonic leading edges and arbitrary, time-dependent downwash distribution. The actual distribution is represented approximately by dividing the wing planform into rectangular sections, in each of which the downwash is assumed uniform. The coefficients can be superposed to determine the aerodynamic forces in gust, stability and flutter problems.

M. Holt, USA

**3162. Richardson, J. R., A method for calculating the lifting forces on wings (unsteady subsonic and supersonic lifting-surface theory),** Aero. Res. Counc. Lond. Rep. Mem. 3157, 30 pp., 1960.

This method is essentially the same as the method developed at the NACA by Charles Watkins, except for a slightly different choice of points at which the calculations are made. Reviewer notes a five-year time-lag between preparation and publication of this report. At the time the work was done it was a useful, original contribution. Now it duplicates work already available in the literature.

L. Trilling, USA

**3163. Pittoni, M., A new theory for triangular swept wings (Theory of the virtual Mach number)** (in Italian), *Aerotecnica* **39**, 5, 239-249, Nov. 1959.

The quantitative and qualitative differences between theoretical and experimental results for triangular swept wings led the author to the idea of revising the classical concept of Mach number for such wings.

Assuming that the operating speed is the effective wing speed (instead of its component normal to the leading edge), author defines in case of subsonic leading edges, the virtual Mach number $M_i$: $M_i^2 = M_0^2 + \mathrm{ctg}^2\, \varepsilon\, (\cos\alpha + \sin\alpha\, \mathrm{ctg}\, \mu_0)^2$, $\varepsilon$ and $\alpha$ being the semiangle of the wing and its incidence respectively, while $M_0$ is the classical Mach number and $\mu_0$ the corresponding semiapex angle. For the case of supersonic leading edges, the same concept $M_i$ is admitted in a first approximation.

A series of applications are then presented (where $M_0$ in the classical formulas is replaced by $M_i$) and compared with the experimental results, a good agreement being obtained.

Reviewer agrees with author on the necessity of extending the idea of virtual Mach number by a more detailed physicomathematical analysis of the concept.

M. I. Ionescu, Roumania

**3164. Margolis, K., and Elliott, Miriam H., Theoretical calculations of the pressures, forces, and moments due to various lateral motions acting on tapered sweptback vertical tails with supersonic leading and trailing edges,** NASA TN D-383, 126 pp., Aug. 1960.

A systematic delineation of formulas for the (12) theoretical stability derivatives is applied to the preparation of graphs, useful for estimating the forces and moments for a given family of sweptback, tapered (supersonic design) vertical tails.

Tabulated taper ratios from 0 to 1.0 (every 0.2) in the design-type charts feature the computations of major parameters in relation to the motions of constant sideslip, lateral acceleration, steady yawing and rolling (with various values of Mach number and aspect ratios), thus offering a readily available preliminary design tool.

The extensive mathematical derivations (about 115 equations) provide an exhaustive coverage for the subject application of the linearized thin-airfoil theory for supersonic speeds (8 appendices). The interrelationships between the postulated parameters and their stability derivatives are given for various (supersonic) Mach numbers and tail geometry parameters.

The research information contains five (5) major references for further correlation and optimization efforts.

C. R. Bell, USA

**3165. Carafoli, E., and Mateescu, D., Supersonic flow around a conical wing-fuselage system** (in French), *Rev. Mécan. Appl.* **4**, 3, 377-390, 1959.

Conical flow past a triangular wing mounted on a cone with apex coincident in a uniform supersonic stream is investigated on the basis of the linear theory; the same problem has already been considered by Browne, Friedman and Hodes [*J. Aero. Sci.* **15**, 1948] on the hypothesis that the angles of attack of the body and of the wing were the same. Authors now consider different values of said angles and give the solution in closed form. Authors use Busemann's transformation for the conical flows: the components of the perturbed velocity are harmonic functions of suitable variables $\eta$, $\zeta$. Applying a conformal transformation $(\eta, \zeta)$ to the plane, authors reduce the problem of the determination of the flow to the one of obtaining a function the imaginary part of which assumes given values on a segment of the real axis, while its real part is null on the rest of said axis.

C. Ferrari, Italy

**3166. Wilby, P. G., An experimental and theoretical investigation of second-order supersonic wing-body interference, for a non-lifting body with wings at incidence,** *Flygtekn. Försöksanst. Medd.* 87, 16 pp., Oct. 1960.

Pressure distributions on the wing of two wing-body combinations are measured experimentally at Mach numbers 3 and 4 with the wing at various incidences in the range 0° to 10°. The results are compared with theoretical results which include interference effects calculated according to the second-order supersonic wing-body interference theory due to Landahl and Beane. This theory, having been tested previously for non lifting wing-body combinations, is thus tested also for wings at incidence. The agreement between theory and experiment is found to vary with Mach number and wing sweepback. For the higher Mach number and moderate sweepback the theory gives a good prediction of pressure distribution, but for the most adverse condition of low

Mach number and large sweepback the theory is found to overestimate the interference effects. This is expected as the theory assumes the sweepback of the wings is small compared with that of the Mach line. An empirical guide to the limit of application of the interference theory is given. Within this limit the agreement between theory and experiment is found to deteriorate only a little with increase of incidence, over the range tested.

From author's summary

3167. **Fournier, P. G., Aerodynamic characteristics at low speed of a reentry configuration having rigid retractable conical lifting surfaces,** NASA TN D-622, 26 pp., Nov. 1960.

Results were obtained through an angle-of-attack range from $-4°$ to $90°$ on a reentry vehicle having a rigid retractable modified conical delta wing of aspect ratio 1.2, leading-edge sweep of $73.2°$, and retractable tail surfaces which could be deflected from $5°$ to $-45°$. Lateral tests were made at an angle of attack of $0°$ through an angle-of-sideslip range of $-12°$ to $32°$.

From author's summary

3168. **Blakeslee, D. J., Empirical relationships for jet-flap lift and drag prediction,** *Aerospace Engng.* **19**, 11, 12-15, Nov. 1960.

Empirical relations for the prediction of the lift and drag of unswept three-dimensional jet-flapped wings are shown, along with the necessary inputs. Plotted data show that the equations organize lift and drag data with a consistency sufficient for preliminary design use in the ranges $1 \leq C\mu \leq 50$, $3 \leq A\!\!\!R \leq 10$.

From author's summary

3169. **Zhigulev, V. N., and Zhilin, Iu. L., On bodies of minimum wave drag,** *Appl. Math. Mech.* (*Prikl. Mat. Mekh.*) **23**, 6, 1462-1475, 1959. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

Authors attack the following problem: find the body possessing the minimum wave drag at supersonic speeds, and having certain properties (like the volume, the lift force, moment, etc.) fixed. The problem is solved for a linearized flow. The general philosophy is as follows: Authors find the envelope of all the characteristic surfaces that separate the perturbed and unperturbed parts of the flow in the direct and reversed streams (i.e., $V_\infty$ and $-V_\infty$). These envelopes intersect along line $L_s$ and the volume enclosed between, $\Omega_s$, has the area $S$. Authors assume that the problem of determining within $\Omega_s$ a potential $\phi$ satisfying the wave equation with its value $\phi_s$ on $S$ given has a solution. Then an arbitrary quantity (property) $K_i$ connected with the geometric or force properties of the body is of the form $K_i = B_i(\phi_S)$, [I], with $B_i$ denoting an integro-differential operator. The variational problem is formulated as that of determining $\phi_s$ corresponding to the flow past a body of minimum drag under condition [I].

Going into the details, authors calculate the volume of the characteristic surface, the force acting on the body, the drag and the expression for the minimum drag. Since the problem is equivalent to the determination of the minimum of a functional, the standard technique of the calculus of variation is used, furnishing the Euler-Lagrangian equation of the problem. The final formula permits calculation of the volume of the unknown body of minimum drag. As an example this philosophy is applied to a body of revolution with a cylindrical duct possessing minimum external drag. As the second example, authors investigate a combination of bodies having minimum wave drag, i.e., a fuselage and a wing. The paper is a nice example of work of so-called mathematical engineers, a type of engineers unknown in U.S.A., but common in Russia.

M. Z. v. Krzywoblocki, USA

3170. **Curry, T. B., Jr., and Crabill, N. L., Rocket-model investigation of lateral stability characteristics and power effects of a jet-engine airplane configuration with tail boom at Mach numbers from 1.15 to 1.37,** NASA TN D-638, 29 pp., Jan. 1961.

A flight test was made to determine the jet effects on the lateral stability of a single-engine airplane configuration. The configuration employed sweptback wing and tail surfaces. The horizontal tail was located ahead of the jet exit and the vertical tail was mounted on a tail boom extending above and behind the jet exit. The jet was provided by a rocket simulating a typical jet engine with afterburner operating. No appreciable power effects were indicated by the static and dynamic lateral stability derivatives derived from the motions or by the pressures measured on the vertical tail.

From authors' summary

3171. **Neihouse, A. I., Klinar, W. J., and Scher, S. H., Status of spin research for recent airplane designs,** NASA TR R-57, 54 pp., 1960.

This report presents the status of spin research for recent airplane designs as interpreted at the Langley Research Center of the National Aeronautics and Space Administration. Major problem areas discussed include:

(1) Interpretation of results of spin-model research
(2) Analytical spin studies
(3) Techniques involved in obtaining measurements of various parameters in the spin
(4) Effectiveness of controls during spins and recoveries
(5) Influence of long noses, strakes, and canards on spin and recovery characteristics
(6) Correlation of spin and recovery characteristics for recent airplane and model designs.

Analyses are made of the existing problems and general conclusions are drawn.

From authors' summary

3172. **Press, H., and Steiner, R., An approach to the problem of estimating severe and repeated gust loads for missile operations,** NACA TN 4332, 44 pp., Sept. 1958.

Paper presents a simplified description of atmospheric turbulence environment derived from airplane measurements. This description is then applied in developing an approach to the estimation of severe and repeated gust-loads. It is of particular interest that the gust environments in storm and nonstorm turbulence are treated separately. Modifications to the atmospheric environment data obtained from airplane surveys are thus possible to account for the effects of the storm-avoidance procedures normally followed in airplane operations.

The authors carefully point out that the procedure outlined in the paper is applicable to missile operations in flight paths similar to those of airplanes. Most missile missions, however, involve nearly vertical boost and reentry flight path. For these cases, the assumption of local isotropy in atmospheric turbulence, which is the basis of the present paper, becomes highly questionable. In addition for a nearly vertically rising missile system, disturbances due to wind shear are generally much more severe than those due to atmospheric turbulence. The responses of such missiles to atmospheric turbulence are insignificant unless the guidance and control system is particularly sensitive to the frequency bands excited by atmospheric turbulence.

H. Lin, USA

3173. **Hunter, P. A., Kraft, C. C., Jr., and Alford, W. L., A flight investigation of an automatic gust-alleviation system in a transport airplane,** NASA TN D-532, 91 pp., Jan. 1961.

3174. **Shenstone, B. S., Engineering aspects in man-powered flight,** *J. Roy. Aero. Soc.* **64**, 596, 471-477, Aug. 1960.

# Vibration and Wave Motion in Fluids

(See also Revs. 2975, 2988, 3080, 3084, 3095, 3225, 3270, 3277, 3279, 3280, 3282, 3303, 3304, 3307, 3308, 3309, 3310, 3356)

**3175. Phillips, O. M., On the dynamics of unsteady gravity waves of finite amplitude: Part 1, The elementary interactions,** *J. Fluid Mech.* **9**, 2, 193–217, Oct. 1960.

Author formulates the beginning of a theory dealing with nonlinear interactions of a random field of gravity waves. Part I deals with interactions of pairs and triads of wave components and their relation to energy transfer. Assuming irrotational motion, Laplace's equation is solved and solution is given in terms of a Fourier-Stieltjes transform, reason being (a) basic equations derived here are easily applicable to random sea, (b) Fourier-Stieltjes transform, representing physical quantities say, are easily observable experimentally. Analysis shows that secondary and tertiary interactions have considerable effect on energy transfers. As an application the interaction of a swell and a local storm is discussed.

Reviewer would like to see (a) justification of assumption of irrotationality, particularly as applied to interaction of a swell and a storm, (b) conditions under which certain series converge. Since the paper is theoretical some mathematical maturity is expected of the reader due to extensive use of Fourier-Stieltjes transforms.

Reviewer does not see an immediate application of the results to engineering. D. Boyanovitch, USA

**3176. Meyer, R. E., On the spreading of surface waves,** Brown Univ., Div. Appl. Math. TR 37 (Contract Nonr 562(07)), 27 pp., Sept. 1960.

The radial spreading of the head of a gravitational wave into water at rest and uniform depth is analyzed on the basis of the shallow-water theory. Predictions are derived for the amplitude decay, the wave distortion and the breakdown of the shallow-water conditions due to bore-formation. Conditions under which the shallow-water theory may be justified are given, and the range of validity of the approximation is discussed. Finally author presents the physical interpretation of the mathematical wave structure. From author's summary

**3177. Yih, C.-S., Gravity waves in a stratified fluid,** *J. Fluid Mech.* **8**, 4, 481–508, Aug. 1960.

This paper summarizes and unifies many results about small amplitude gravity waves in stably stratified fluids with or without density discontinuities. The results are not new but some are given greater generality. The paper is useful in that many results, obscured in treatments by other authors of particular problems (e.g. meteorological, in which connection the list of references is quite inadequate), are now treated together. Many have been treated using different forms of words in the meteorological literature with which this author appears to have been unfamiliar at the time of writing.

The fluid is inviscid and the usual perturbation equations used. The phase velocity is increased by increase in wavelength and reduced by compressibility. Two types of wave, internal and surface, are discussed, latter not always faster if stratification intense enough; there is no interference of different modes and equipartition of energy holds; results are generalized to many layers; effects of a wave machine are formulated, and instability due to vertical oscillations of bed exciting resonant stationary waves at surface are discussed.

Too much is made of a special case in a semi-infinite fluid in which the motion is along isobars and of waves with sonic speed in a compressible fluid with a free surface, but the treatment is very competent and possesses unusual sparkle which the author has since developed. The reference to hydraulic jumps in a heterogeneous fluid could do with elucidation; they have never been defined in these circumstances because they are not unique. R. S. Scorer, England

**3178. Miles, J. W., The hydrodynamic stability of a thin film of liquid in uniform shearing motion,** *J. Fluid Mech.* **8**, 4, 593–610, Aug. 1960.

Author gives an approximate determination, for large values of the Reynolds number, for the conditions governing surface wave formation on a thin film of liquid that is bounded below by a wall and subjected to a prescribed shearing stress at its upper and otherwise free surface. The flow is supposed incompressible, two-dimensional and laminar. Problem has already been studied by Feldman [AMR **10**(1957), Rev. 3709], but author makes some criticism of Feldman's paper. The analysis is similar to that for plane Couette flow and eigenvalue equation is considered also for the inviscid problem. Author concludes that: (a) thin film of liquid is unstable for sufficiently large values of the Reynolds number ($R$) and the Weber number ($W$); (b) sufficient condition of stability is either $R < 203$ or $W < 3$.

One appendix gives a table for the values of the function $F(z)$, previously calculated by Lin [AMR **9**(1956), Rev. 1491]. G. Sestini, Italy

**3179. Rosciszewski, J., Propagation of waves of finite amplitude along a duct of non-uniform cross-section,** *J. Fluid Mech.* **8**, 4, 625–633, Aug. 1960.

The simple wave in a duct of nonconstant cross section is treated by means of a perturbation technique, which uses the solution for constant cross section as lowest-order approximation. K. G. Guderley, USA

**3180. Escande, L., Remenieras, G., and Claria, J., Speed of the waves and overpressures in constrained pipes** (in French), *C. R. Acad. Sci. Paris* **251**, 3, 305–307, July 1960.

**3181. Snegirev, I. A., Hydraulic jump in a channel having an opposite bottom slope** (in Russian), *Gidrotekh. Stroit.* **30**, 4, 49–50, Apr. 1960.

Formula for the hydraulic jump in inclined channel contains an area of a longitudinal section (the volume) of the nonunifom portion of flow. Empirical relationship with the Froude number before the jump is established from 120 experiments at various adverse slopes, from 0 to 0.20, and an approximate formula deduced. Two simple formulas for the height and length of the jump are proposed as improvements of similar values in channels having horizontal bottoms. S. Kolupaila, USA

**3182. Lunyakina, T. B., The effect of friction on the ordinate of a straight water hammer effect** (in Russian), *Trudl Tbillisk. In-ta Inzh. Zh.-d. Transp.* no. 31, 82–96, 1957; *Ref. Zh. Mekh.* no. 9, 1959, Rev. 10109.

The starting point adopted by the author is the approximate expression for the general integral of the differential equations for an elastic water hammer having small linear hydraulic resistances

$$\begin{aligned} y - y_\bullet &= [F(x - at) + f(x + at)]\, e^{-\mu t} \quad \left(\mu = \frac{\lambda v_\bullet}{4D}\right) \\ v - v_\bullet &= -\frac{g}{a}[F(x - at) - f(x + at)]\, e^{-\mu t} \end{aligned} \qquad [1]$$

(the symbols used are universally recognized). In the case of a simple pipe-conduit with a length $L$ and a straight water hammer the following equations arise from equation [1]

$$\mu = \frac{1}{\tau} \log_e \frac{(a/g)v_\bullet + y(t - \tau) - y_\bullet}{(a/g)v_\bullet - y(t) + y_\bullet} \qquad \left(\tau = \frac{2L}{a}\right) \qquad [2]$$

where $y(t)$ is the value of $y$ at the slide plate at the end of the pipe conduit. However, the linearization leading to equation [1] is only permissible with small divergencies of velocities of the flow in relation to $\nu_0$ and not for a straight water hammer when the velocity of flow at the end of the pipe conduit changes from $\nu_0$ to 0, and to start with, fluctuates between $\nu_0$ and $-\nu_0$. The calculation of $\mu$ according to [2] is fulfilled on the basis of experiments on two sets of apparatus with a diameter for the tubes of 32 mm and lengths of 12.9 and 54.2 m, with initial velocities for the flow of 2.6 to 108 cm/sec, that is to say of Reynolds' numbers ~ 600 to 27,000, which correspond to a laminar regime and a transition region. Some other experiments on the direct water hammer were also made use of. Analysis of the tests enabled the author to scrutinize the linear link between

$$\lg \mu \frac{D}{2\nu_0} \lg \frac{\lambda}{8} \quad \text{and} \quad \lg q = \lg \frac{a\nu_0}{2gy_0}$$

though the experimental points in the corresponding coordinates show extreme divergencies. Noting the strongly marked increase of the value for $\lambda$ in the transient regime in comparison with the values in the steady regime the author suggests as a possible cause of this increase, taken together with differences in the field of velocities, the penetration into the tube of a certain amount of air through the joints. Since the regimes being investigated are situated in the laminar and transition regions it is impossible to assert with any emphasis that the relations of $\lambda$ to $q$ which have been discovered are linked up with the dynamic phenomena in the tube; these might well be the consequence of the relation of $\lambda$ to $R$ in the stationary regime.

N. A. Kartvelishvili

*Courtesy Referativnyi Zhurnal, USSR*

**3183. Escande, L., and Claria, J., Overpressure in the case of a surge tank** (in French), *C. R. Acad. Sci. Paris* **250**, 26, 4243-4245, June 1960.

**3184. Sideriades, L., Free oscillations in the surge tank** (in French), *C. R. Acad. Sci. Paris* **250**, 26, 4277-4279, June 1960.

**3185. Sideriades, L., Surge tanks: Non-linear study of the equation $Q(H + Z) = Cte$** (in French), *C. R. Acad. Sci. Paris* **250**, 25, 4102-4104, June 1960.

**3186. Escande, L., Rhythmic movements in the case of a surge tank with the influence of the height of the turbine and head** (in French), *C. R. Acad. Sci. Paris* **250**, 20, 3252-3255, May 1960.

**3187. Sideriades, L., Stability conditions of hydraulic systems with two surge tanks** (in French), *C. R. Acad. Sci. Paris* **249**, 23, 2486-2488, Dec. 1959.

**3188. Loginov, V. N., A dynamic method for computing the strength of protective works when under the impact of waves** (in Russian), *Trudi Tsentr. Nauk-i. In-ta Morsk. Flota* no. 19, 58-68, 1958; *Ref. Zh. Mekh.* no. 9, 1959, Rev. 10596.

An analysis is carried out on the action of broken-up waves on a mole of vertical profile, and a method is proposed for the calculation of the maximum "give" of the structure under the action of a wave load. The rotational vibrations of the given solid body round the principal axes of vibration are investigated in lieu of examination of the progressive vibration of the body in the horizontal direction and the rotational vibration of the body round the center of inertia. When determining the frequencies of the natural vibrations of the wall round the axes of vibration consideration is given to the influence exerted by the mass of the water moving with the wall.

K. M. Tuchina

*Courtesy Referativnyi Zhurnal, USSR*

**3189. Thirriot, C., On the phenomena of shock-wave propagation in the discharge gallery of a subterranean hydroelectric plant** (in French), *C. R. Acad. Sci. Paris* **249**, 25, 2716-2718, Dec. 1959.

**3190. Clenshaw, C. W., and Elliott, D., A numerical treatment of the Orr-Sommerfeld equation in the case of a laminar jet,** *Quart. J. Mech. Appl. Math.* **13**, 3, 300-313, Aug. 1960.

Another interesting technique: By expanding even disturbance function, after coordinate transformation $t = \tanh y$, in Chebyshev polynomials $T_n(t) = \cos(n \cos^{-1} t)$, Orr-Sommerfeld equation yields five-term recurrence relation between coefficients suitable for numerical iteration. Upper limb of neutral stability curve agrees reasonably well with results of Curle and of Tatsumi and Kakutani. Near critical Reynolds number, 96 terms are used on NPL DEUCE, yielding critical value of about 3.7, comparing favorably also with Howard. Reviewer feels parallel flow assumption is poor for practical jets at such low Reynolds numbers, and R. C. Chaunaud [M. S. Thesis, University of California, Los Angeles, 1960] found pseudo-laminar theory inadequate to explain his experimental results. But pseudo-laminar jet instability will continue to be a useful challenge for mathematical techniques of handling the Orr-Sommerfeld equation.

A. Powell, USA

**3191. Smith, D. H., and Wintle, H. J., The propagation of sound in relaxing gases in tubes at low frequencies,** *J. Fluid Mech.* **9**, 1, 29-38, Sept. 1960.

Theoretical and experimental investigation of the velocity of propagation and the attenuation of plane sound waves in gases which undergo vibrational relaxation. The theory presented shows that, at low audible frequencies, the velocity is the relaxation velocity diminished by the Helmholtz-Kirchhoff correction, while the tube absorption due to viscosity and thermal conduction and the relaxation absorption in the gas are additive, no interaction taking place. These relations have been fully confirmed experimentally, the experimental Helmholtz-Kirchhoff coefficients being about 15% in excess of the calculated values.

Standing-wave measurements were made at frequencies from 80 to 1500 cycles/sec, using an acoustic interferometer 175 cm long and of 3.78 cm internal diameter. Pure $CO_2$, dry air free from $CO_2$, and $O_2$ with an impurity of about 2 parts in $10^7$ were used. The measured values were then reduced to the standard conditions of 1 atm and 30 °C. The free gas velocities of sound waves, extrapolated to zero frequency, were obtained as 270.57 ± 0.04, 349.18 ± 0.02, and 331.33 ± 0.04 meter/sec.

By use of the Beattie-Bridgman equation of state, the molar heat capacities $C_p$ and $C_v$ and the specific heat ratio $\gamma$ were determined at 1 atm and 30 °C. For $CO_2$, $C_p/R = 4.537 \pm 0.008$ and $\gamma = 1.2907 \pm 0.0004$, while for $O_2$, $C_p/R = 3.547 \pm 0.003$ and $\gamma = 1.3954 \pm 0.0003$. On extrapolation to zero pressure these values become, with unchanged uncertainties, 4.509, 1.2849, 3.542, and 1.3934.

R. Heller, USA

**3192. Benney, D. J., and Lin, C. C., On the secondary motion induced by oscillations in a shear flow,** *Physics of Fluids* **3**, 4, 656-657 (Letters to the Editor), July/Aug. 1960.

## Fluid Machinery

(See also Revs. 2870, 3043, 3202, 3207)

**3193. Stewart, W. L., Whitney, W. J., and Wong, R. Y., A study of boundary-layer characteristics of turbomachine blade rows and their relation to overall blade loss,** *ASME Trans.* **82** D (*J. Basic Engng.*), 3, 588-592, Sept. 1960.

Authors derive over-all turbomachine blade row loss, from boundary-layer momentum thickness, trailing-edge thickness, solidity, outlet angle. Momentum thickness is correlated against Reynolds number and total surface diffusion, and surface diffusion is determined as function of blade row solidity and reaction, assuming Zweifel's form of blade loading (suction and pressure-surface velocities constant across chord). Over-all loss can then be expressed as function of blade geometry (inlet, outlet angles and solidity).

Authors have provided information useful to designers, but validity depends upon whether momentum thickness-diffusion factor correlation is universal. Further data on this correlation are required.

J. H. Horlock, England

**3194. Gallant, H., The cascade correction due to Betz** (in German), *Öst. Ing.-Arch.* **13**, 4, 264–267, 1960.

A. Betz developed in 1931 an approximate method to determine the deviation of the flow pattern produced by a cascade of equal wing profiles at equal distances, when the characteristic properties of the profile in parallel flow are known. This method saves considerable work compared with the classical conformal mapping or the singularity method of Schlichting. Author improves Betz's method and also takes the thickness of the profile into consideration using calculations of B. Eckert on Joukowsky profiles. Numerical results are given for cascades with the profile Goe. 389.

K. Pohlhausen, USA

**3195. Staniforth, R., Some tests on cascades of compressor blades fitted with vortex generators,** Aero. Res. Counc. Lond. Curr. Pap. 487, 18 pp., 1960.

Cascade tests showed that the fitting of vortex generators to the convex surface of compressor blades increased the two-dimensional losses under all conditions. A more detailed investigation indicated that the vortex generators were, in fact, suppressing shock-induced flow separation, but that boundary-layer separation still occurred at about 50 per cent chord–probably due to the increased boundary-layer thickness produced by the vortex generator drag.

While there is always a possibility that some different configuration will be successful, the scheme of increasing the drag critical Mach number of a compressor cascade by means of vortex generators is, on the basis of available test evidence, a failure.

From author's summary

**3196. Romanenko, P. A., Approximate analysis of axisymmetric flow past a fixed cascade stage with cylindrical boundary in axial turbomachine** (in Russian), *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk* no. 6, 68–78, Nov./Dec. 1959.

Author presents a method of approximate calculation of three-dimensional flow past a fixed cascade of axial turbomachines. The method is based on the following fundamental assumptions: (1) Steady, axisymmetric flow of inviscid fluid. The average values of the parameters of the actual flow taken across one pitch are considered as parameters of the substitute flow; (2) cylindrical boundary of the flow section of the stage; (3) the generatrices of the blades are purely radial; (4) at an infinite distance upstream from the cascade the pressure and velocity along the radius are constant and the flow is purely axial.

By introducing the stream function the author converts the calculation to the solution of a nonlinear partial differential equation of the elliptic type with variable coefficients. First he treats the case of incompressible fluid for which this equation changes into a second-order linear equation of the elliptic type with variable coefficients. The Galerkin method is suggested for the solution of this equation. The effect of compressibility is then investigated by the method of subsequent approximations. The numerical procedure is illustrated by an example.

J. Polasek, Czechoslovakia

**3197. Krzywoblocki, M. Z., Compressibility effects in circumferential inlet distortion in axial compressors: Parts 1 and 2** (in English), *Öst. Ing.-Arch.* **13**, 4, 214–235, 1959; **14**, 2, 79–92, 1960.

Part I deals with the application of similarity laws to theory of Ehrich [*J. Aero. Sci.* **24**, 413–417, 1957] to allow it to be applied to compressible subsonic flows. In Part II, a similar analysis is applied to the theory of Yeh [G. E., Aircraft Gas Turbine Div., no. R57AGT628, Sept. 1957] and the method of solution is outlined for two particular examples. No numerical data are shown.

S. R. Montgomery, England

**3198. Gut, W., Comparison of the relative losses and characteristics of axial-flow compressor stages,** *Escher Wyss News* **32**, 2/3, 3–13, 1959.

The problem of optimum velocity triangles for multi-stage axial flow compressors is treated. First, on a two-dimensional basis, assuming the same lift-drag ratio for stator and rotor blades, the losses are plotted as a function of the flow coefficient with the reaction ratio $R$ as a parameter. Similar relations are derived for a hub-tip ratio smaller than one, assuming free vortex flow blading.

Tests show a marked deviation from the analytical result in the sense that stages with more than 100% reaction (this corresponds to acceleration in the stator) have efficiencies which are equal to the efficiencies in stages with 50% reaction ratio (symmetrical velocity triangles) which, according to the analysis, would be optimum. This observation is interpreted by an increased lift-drag ratio in the $R > 100\%$ design.

It is further shown that such a design ($R > 100\%$) has other advantages if no Mach number limitation exists—specifically, broader range.

This reviewer thinks that comparison with data of NACA Reports RM-E56BO3, RM-E56BO3a, and RM-E56BO3b, axial flow compressor design handbooks would be interesting.

H. P. Eichenberger, USA

## Flow and Flight Test Techniques and Measurements

(See also Revs. 3098, 3159, 3171, 3260, 3287, 3288, 3364)

**Book—3199. Troskolanski, A. T., Hydrometry; Theory and practice of hydraulic measurements,** New York, Warsaw; Pergamon Press and Państwowe Wydawnictwa Techniczne, 1960, xx + 684 pp. $16.80.

This is a revised and enlarged translation from Polish "Hydromechanika Techniczna, Vol. III, Pomiary wodne, 1957," favorably evaluated in AMR 11(1958), Rev. 502. Book contains principles and methods of measurement of hydromechanical quantities, measuring instruments and apparatus used in hydraulic measurement, and hydrometric laboratories. First, author reviews all kinds of liquid measurements: time, angle, height, surface, volume, pressure, velocity, and intensity of flow, which he distinguishes from the total flow. Hydrometric instruments are presented separately, for each purpose. Both types of current meters, European and American, are discussed and illustrated. All other methods, such as weirs, orifices, flumes, tracer and other physical methods are mentioned and described also. Greatest emphasis is laid on water meters, as this field is most familiar to the author, and this is undoubtedly the most authoritative part of the book. Hydrometric laboratories are understood for water-meter testing only, although they could include all measurements in hydraulic laboratories. Reviewer diagrees with inclusion of pendulum devices into floats, as tilting floats; they should belong rather to dynamometers.

Author and editors are to be congratulated with accomplishment of an important task. A particular significance of this book is that

it bears the title "Hydrometry" for the first time in the English language. It will be well accepted by our hydraulic engineers.

S. Kolupaila, USA

**3200. Shaw, R., The influence of hole dimensions on static pressure measurements,** *J. Fluid Mech.* **7**, 4, 550–564, Apr. 1960.

In this article the difference, $\Delta p$, between the static pressure measured by a piezometric hole, and the true pressure in incompressible fluid is investigated in uniform flow through smooth circular pipes. The value of $\Delta p$ is determined experimentally, special care being taken of the accuracy of the measuring device (e.g. the elaboration of the edges near the hole was controlled far below 0.01 mm). The results are presented in form of a series of diagrams, connecting $\Delta p/\sigma_0$ ($\sigma_0$ being the shear stress near the wall) with the Reynolds number, *Re* (characterized by the shear-velocity and hole diameter $\sigma$). The $l/d$ ratio is varied, too, where $l$ is the hole depth. In all cases it was found that $\Delta p/\sigma_0$ is positive and that it increases with $l/d$ and *Re*, showing a tendency to a constant value for $l/d > 1.5$ and $Re > 700$. This limit value seems to be $\Delta p/\sigma_0 = 2.8$. The influence of burrs in the downstream edge of the holes was found to be very great and their value of about 0.05 mm may cause an error several times as large as in the case of a wholly smooth hole.

M. M. Boreli, Yugoslavia

**3201. Kalinske, A. A., The twin throat venturi: A new fluid-flow measuring device,** *ASME Trans.* **82** D *(J. Basic Engng.)*, 3, 710–716, Sept. 1960.

Author describes design and performance of twin-throat venturi—a new venturi having a length roughly equal to its inlet diameter. The device has low relative head loss, comparable to long-form venturi tube, and less than losses occurring in short-form venturi tube. Basic idea of twin-throat venturi is a second throat consisting of a curved profile and large portion of pressure differential is caused by flow over a curvilinear surface. Discharge coefficient for device is constant above some critical Reynolds number, which is a function of $\beta$ (throat-to-pipe diameter). Constant value of discharge coefficient ranged between 0.60 and 0.68, depending on value of $\beta$.

J. G. Knudsen, USA

**3202. Lee, W. F. Z., and Karlby, H., A study of viscosity effect and its compensation on turbine-type flowmeters,** *ASME Trans.* **82** D *(J. Basic Engng.)*, 3, 717–728, Sept. 1960.

Authors present a preliminary analysis of the viscosity effect on accuracy of axial-flow turbine meter handling viscous fluids. Expressions for the meter registration (ratio of angular velocity of rotor to volumetric flow rate) are derived for the laminar, transition, and turbulent boundary layers on the turbine blades. Viscosity has large effect in laminar and transition region. Viscosity effect in transition region is analyzed and mechanical device proposed to compensate for it. A prototype containing the compensating device was tested on two fluids (viscosity 100 and 250 SSU). The viscosity compensator makes the meter output independent of viscosity in the transition region.

J. G. Knudsen, USA

**3203. Bogema, M., and Monkmeyer, P. L., The quadrant edge orifice—A fluid meter for low Reynolds numbers,** *ASME Trans.* **82** D *(J. Basic Engng.)*, 3, 729–734, Sept. 1960.

Experiments indicate that discharge coefficient is constant (± 0.5%) for pipe Reynolds numbers greater than about 3000 to 5000, depending on diameter ratio of orifice. Upper limit of constant coefficient exceeds in most cases the maximum that could be reached with available equipment (25,000 to 100,000). Good reproducibility of tests with respect to installation and construction of orifice is found. In discussion, D. J. Evans points out that discharge coefficient increases rapidly at Reynolds numbers near 125,000, so that range of constant coefficient cannot be extended by extrapolation, and he describes a design modification that may extend range of constant coefficient beyond Reynolds number of 200,000.

G. Rudinger, USA

**3204. Filban, T. J., and Griffin, W. A., Small-diameter-orifice metering,** *ASME Trans.* **82** D *(J. Basic Engng.)*, 3, 735–740, Sept. 1960.

Experimental setup is described to test carefully constructed three-quarter and one-inch meters. A general expression is developed for the flow coefficient as a function of diameter ratio and Reynolds number. Discussion deals with reliability of corner taps, used in the experiments, possible effects of vibration, and alternative forms of presentation of the results.

G. Rudinger, USA

**3205. Walters, K., The motion of an elastico-viscous liquid contained between concentric spheres,** *Quart. J. Mech. Appl. Math.* **13**, 3, 325–333, Aug. 1960.

Using a tensorial constitutive equation due to Oldroyd [title source **4**, p. 271, 1951; AMR **5**(1952), Rev. 1075] oscillatory flow is analyzed for a hypothetical elastoviscometer in which (from the author's summary) "the outer of two concentric spheres undergoes forced harmonic angular oscillations about a vertical axis and the inner sphere is constrained by a torsion wire. It is concluded that such an instrument would distinguish different elastoviscous liquids in much the same way as the coaxial-cylinder elastoviscometer."

S. Corrsin, USA

**3206. Ashwin, B. S., Hagyard, T., Saunders, I. C. B., and Young, T. E. Viscometers having damped torsional oscillation,** *J. Sci. Instrum.* **37**, 12, 480–485, Dec. 1960.

Two types of torsional pendulum viscometers, having only one solid-fluid boundary, have been developed specifically for use in fluidized solids investigations. They may have utility also in coarse or fibrous suspensions or in melts at high temperatures. The theory of these viscometers is discussed and empirical correlations are presented whereby any instrument of these two types may be used with minimal or no calibration.

From authors' summary

**3207. Wesko, J. R., and Kageyama, Y., Investigation of the flow through axial flow compressor rotor blades by means of the electric spark technique,** AFOSR 212 (Univ. Maryland, Inst. Fluid Dynam. Appl. Math. TN BN 228), 11 pp. + figs., Jan. 1961.

Instantaneous flow patterns of subsonic flow past the blades of single-stage axial flow compressor rotor were derived from multiple periodic spark traces. Processes of evaluation and of analysis were developed by which the characteristics of performance may be related to the particulars of interaction between the flow and the rotating blading system. Possibilities of extension of the technique of investigation to transonic and supersonic flow and to highly three-dimensional patterns are indicated.

From authors' summary

**3208. Van Der Walle, F., Theoretical determination of the power efficiency and overall flow behaviour of free jet wind tunnels with special emphasis on transonic wind tunnels** (in English), Nat. Luchtlab. Amsterdam Rep. G. 1, 12 pp., Dec. 1957.

The theory presented for the characteristics of a free jet tunnel agrees well with experimental data. The theory appears to be applicable to other types of tunnels. The use of a loss factor shows a blocking phenomenon in free jet tunnels that is analogous to the blocking phenomenon found in closed-wall tunnels.

R. C. Binder, USA

**3209. Spreiter, J. R., Smith, D. W., and Hyett, B. Jeanne, A study of the simulation of flow with free-stream Mach number 1 in a choked wind tunnel,** NASA TR R-73, 36 pp., 1960.

The degree to which experimental results obtained under choking conditions in a wind tunnel with solid walls simulate those associated with an unbounded flow with free-stream Mach number 1 is investigated for the cases of two-dimensional and axisymmetric flows. It is found that a close resemblance does indeed exist in the vicinity of the body, and that the results obtained in this way are generally at least as accurate as those obtained in a transonic wind tunnel with partly open test section. Some of the results indicate, however, that substantial interference effects, particularly those of the wave reflection type, may be encountered under certain conditions, both in choked wind tunnels and in transonic wind tunnels, and that the reduction of these interference effects to acceptable limits may require the use of models of unusually small size.
From authors' summary

**3210. Armstrong, J. C., Flados, R. W., and McKenzie, W. T., Simulated flight testing of a cowled turbojet engine by use of an exhaust ejector,** *Aerospace Engng.* **19**, 11, 16–17, Nov. 1960.

A simple and consequently inexpensive method of testing jet engines and calibrating flight-test instrumentation at simulated flight conditions is presented here. The method was developed and used at the Convair-Fort Worth Ground Test Stand during the B-58 program. Results proved the method to be a useful tool in determining in-flight thrust and analyzing various engine performance parameters.
From authors' summary

**3211. Heyson, H. H., Measurements of the time-averaged and instantaneous induced velocities in the wake of a helicopter rotor hovering at high tip speeds,** NASA TN D-393, 44 pp., July 1960.

Report gives values of instantaneous and time-averaged induced velocities measured beneath a lifting rotor operating at tip speeds as high as 1100 fps. No effects of Mach number on either the root-mean-square velocity fluctuations or the wave shape of the velocity pulses in the wake were noted. Small changes in the wake were measured, apparently resulting from a compressibility effect on the blade load distribution.
A. Gessow, USA

## Thermodynamics

(See also Revs. 2831, 3191, 3249, 3257, 3263, 3269, 3270, 3271, 3302, 3315)

**3212. Coleman, B. D., and Noll, W., On the thermostatics of continuous media** (in English), *Arch. Rational Mech. Anal.* **4**, 2, 97–128, Dec. 1959.

A study of thermal equilibrium states and their stabilities for continuum statics under the action of force and temperature distribution, in terms of phenomenologically defined thermomechanical parameters such as force potential, entropy, free energy, etc.

The work is based on two postulates: (1) any local thermomechanic state can be an equilibrium state provided the local temperature and local force have appropriate values; and (2) at least in continuum, absolute temperature is non-negative. Physical contents of these postulates are primarily the first and second laws of thermodynamics for statics of continuous media.

Local thermal equilibrium and global stability under action of (force-temperature) pair are defined by: (1) vanishing of resultant force and moment, and (2) either minimum energy for (configuration-entropy) state, or maximum entropy for (configuration-energy) state.

From postulates and definitions above, relationships between stress-strain equation and the caloric equation of state, and various inequalities restricting the form of the caloric equation of state are developed and discussed. Paper first explores the consequences for caloric equation of state of the existence of local states of thermal equilibrium, then derives useful, necessary and sufficient criteria for global states to be stable.

On the whole, paper is interesting and well presented, although pedagogical. Reviewer believes there is an error in the definition of Gibb's stability (p. 126), and should be corrected to read:

"An equilibrium state ($f$, $\varepsilon^*$) of a fluid body $B$ is called $G$ stable if any other state [$f^*$, $\varepsilon^*$] with the same total volume and the same total internal energy as [$f$, $\varepsilon$], . . . has a *lower* total entropy, $\int \hat{\eta}[\nu^*(x), \varepsilon^*(x); x]\, dm < \int \hat{\eta}[\nu(x), \varepsilon(x); x]\, dm, \ldots$"
H. S. Tan, USA

**3213. Guiraud, J.-P., On the statistical interpretation of the notion of thermodynamic equilibrium** (in French), *C. R. Acad. Sci. Paris* **250**, 1, 70–72, Jan. 1960.

**3214. Itean, E. C., Glueck, A. R., and Svehla, R. A., Collision integrals for a modified Stockmayer potential,** NASA TN D-481, 29 pp., Jan. 1961.

Collision integrals were calculated for the modified Stockmayer potential $E(r) = 4\varepsilon[(\sigma/r)^{12} - (\sigma/r)^6 - \delta(\sigma/r)^3]$, which may be applied to polar molecules. It was assumed that the colliding molecules maintain their same relative orientation during the encounter. Calculations of the integrals were made for a large reduced temperature range and for a range of $\delta$ from 0 to 10. The results agree with other work on nonpolar interactions ($\delta = 0$). However, for polar interactions, the only previously published calculations (Krieger) have been found to be in error and do not agree with this work.

Assuming that the molecules interact as alined dipoles of maximum attraction, values for $\sigma$, $\varepsilon$, and $\delta$ were determined for various polar molecules by a least-squares fit of experimental viscosity data. Satisfactory results were obtained for slightly polar molecules, but not for more highly polar molecules such as $NH_3$ or $H_2O$. Therefore, it appears that the assumed model of molecules interacting at all times as alined dipoles of maximum attraction is not satisfactory for estimating transport properties of polar molecules.
From authors' summary by E. A. Mason, USA

**3215. Mott-Smith, H. M., Collision trajectories for inverse power intermolecular potentials,** *Physics of Fluids* **3**, 5, 721–724, Sept./Oct. 1960.

**3216. Balian, R., De Dominicis, C., Expression of thermodynamical quantities in terms of distribution of quasi particles** (in French), *C. R. Acad. Sci. Paris* **250**, 25, 4111–4113, June 1960.

**3217. Schaefer, C. A., and Thodos, G., Reduced density correlation for hydrogen: liquid and gaseous states,** *AIChE J.* **5**, 2, 155–158, June 1959.

Correlation is based on study of 57 sources from Amagat (1880) to Johnston, Keller, and Friedman (1954). Check of finished correlation at 485 representative points yielded an average deviation of 0.49% between the correlation and the experimental result. Reviewer found no omissions in sources studied and feels comparison with data is satisfactory. However, description of deviations is not clear; except for over-all comparison, they are not identified as average, standard, or maximum deviations.
R. R. Hughes, USA

**3218. Vaisman, M. D., Some problems in the thermodynamics of one-component vapour-liquid systems** (in Russian), *Trudï Leningrad Politekhn. In-ta* no. 193, 108–118, 1958; *Ref. Zh. Mekh.* no. 9, 1959, Rev. 9624.

**3219. Rant, Z., Exergy diagrams for water and water vapor** (in German), *Brennstoff-Wärme-Kraft* **12**, 7, 297–301, July 1960.

Diagrams for obtaining the exergy (technical work capability) of water and water vapors are presented and two practical examples are calculated with these diagrams. Together with the description of the exergy diagrams for air and combustion gases previously published in this journal, this constitutes the means for carrying out exergetic investigations in steam technique.

From author's summary

**3220. Scala, S. M., A note on the thermal diffusion ratio in dissociated air,** AFOSR Document 21 (Gen. Elect. Co., Space Sciences Lab. R60SD487, Tech. Information Series), 19 pp., Nov. 1960.

**3221. Plank, R., A state equation for liquid water valid up to 300° C and 1200 atm. abs. (17,640 lb/sq in.)** (in German), *Brennstoff-Wärme-Kraft* **12**, 7, 302–304, July 1960.

A semi-empirical state equation for water is presented which reproduces very exactly the latest values for the specific volume of water over the whole range (from 50 to 300°C and from 1 to 1200 atm. abs.), which have been measured in the Moscow Energetic Institute.

From author's summary

**3222. Corruccini, R. J., and Gniewek, J. J., Specific heats and enthalpies of technical solids at low temperatures,** Nat. Bur. Stands. Monogr. 21, 20 pp., Oct. 1960.

**3223. Guiraud, J.-P., On the statistical interpretation of the phenomenologic theory of irreversible thermodynamical processes** (in French), *C. R. Acad. Sci. Paris* **251**, 2, 213–215, July 1960.

**3224. Roy, M., Slightly irreversible transformations and relations of Onsager. Thermic exchanges and passive resistances** (in French), *C. R. Acad. Sci. Paris* **250**, 4, 639–642, Jan. 1960.

**3225. Rosen, G., Entropy production and pressure waves,** *Physics of Fluids* **3**, 2, 188–190, Mar./Apr. 1960.

Neglecting thermal conduction and viscous dissipation, the velocity-free equations of one-dimensional flow are derived with the inclusion of general fluid thermodynamics. The *strength* of finite amplitude pressure waves is introduced into the theory. This quantity is conserved if the entropy of each fluid particle remains constant with time. The growth or decay of the *strength* is examined for a perfect gas which produces entropy.

From author's summary

**3226. Roy, M., On the effluency and production of entropy and on the law of Fourier** (in French), *C. R. Acad. Sci. Paris* **250**, 1, 35–38, Jan. 1960.

## Heat and Mass Transfer

**(See also Revs. 2891, 2966, 3126, 3144, 3147, 3152, 3267, 3270, 3286, 3325)**

**3227. Hall, R. C., Graphical-numerical determination of variable diffusion coefficient in solids,** *Canad. J. Chem. Engng.* **38**, 5, 154–159, Oct. 1960.

Application of Crank-Jost equation for variable diffusivity, which is derived from similarity solution. Approximate upper and lower bound extrapolation are made for deeper depth out of necessity. Sensitivity of result to extrapolation and natural radiation are investigated.

W.-H. Chu, USA

**3228. Holter, W. H., and Grover, J. H., Insulation temperature for the transient heating of an insulated infinite metal slab,** *ARS J.* **30**, 9, 907–908 (Tech. Notes), Sept. 1960.

The solution is given for the equation of transient heat conduction at the hot surface of the insulation of an infinite metal slab. A graphical representation of a simplified approximate solution of sufficient accuracy for engineering use is also given.

From authors' summary

**3229. Lowan, A. N., On the numerical treatment of heat conduction problems with mixed boundary conditions,** *Math. Comput.* **14**, 71, 266–270, July 1960.

Author considers the two-dimensional transient heat conduction problem in a rectangle with prescribed temperature on part of the boundary, $B_1$, and prescribed gradient on remainder of the boundary, $B_2$. The case where part of a side of the rectangle is in $B_1$ and the remainder in $B_2$ has apparently not been treated analytically. Using the simplest explicit finite difference analog of the partial differential equation, author obtains

$$k[\Delta t/(\Delta x)^2 + \Delta t/(\Delta y)^2] \leqslant 1/2$$

as the condition for stability of the numerical solution. Here $\Delta x$ and $\Delta y$ are the space intervals, $\Delta t$ the time interval, and $k$ thermal diffusivity.

It is remarked that the analysis may be extended to the more general boundary condition $pT + q(\partial T/\partial n) = F(t)$, and to problems with cylindrical and spherical symmetry.

H. G. Landau, USA

**3230. Vodicka, V., Steady temperature field in a semi-infinite composite plate,** *Appl. Scient. Res. (A)* **9**, 2/3, 190–196, 1960.

This paper presents a contribution to generalizing classical problems on heat conduction to the case of composite media. The (nesessary) calculations are comparatively complicated.... General deductions are applied to special cases.

From author's summary by J. G. Bartas, USA

**3231. Hatch, J. E., Schacht, R. L., Albers, L. U., and Saper, P. G., Graphical presentation of difference solutions for transient radial heat conduction in hollow cylinders with heat transfer at the inner radius and finite slabs with heat transfer at one boundary,** NASA TR R-56, 49 pp.

This report presents in graphical form the results of computations carried out by the use of the finite-difference method for heat conduction in hollow circular cylinders of infinite length and in infinite slabs. In these graphs transient temperature histories are plotted using the conventional dimensionless groups on a linear scale for a range of position ratios, external to internal diameter ratios and for various values of the natural convection dimensionless group ($bD/k$). The distributions reported are for a heat flux at the inner boundary of the cylinder and at one of the boundaries of the slab while the opposite boundaries are thermally insulated. The ranges of variables reported seem to be adequate for missile calculations where a hollow cylinder or slab is used as heat capacitor.

In the reviewer's opinion the title and abstract of this report are somewhat misleading because of the use of the terms "finite" in both, whereas the treatment is actually for infinite cases.

A. H. El-Waziri, USA

**3232. Chekalev, K. N., and Shumakov, N. V., An approximation for the boundary conditions for processes of nonstationary heat exchange** (in Russian), *Inzhener.-Fiz. Zh.* **1**, 3, 91–94, 1958; *Ref. Zh. Mekh.* no. 9, 1959, Rev. 10243.

An experimental computational method is described for ascertaining the temperature field in a body. The method consists of the solution of the equation for heat conductivity with boundary conditions found by experiment. The boundary function obtained by

experiment is given an approximate value by the sum total of the exponents

$$q(\tau) = \sum_{i=1}^{m} c_i \exp(-b_i\tau)$$

It is shown that the known solution for the one-dimensional equation for heat conductivity with this form of boundary condition of the second order is the solution of the equation with a constant boundary condition of the third order in a regular region. The paper continues by an evaluation of the error caused by disregarding the sum total of a series in the solution. The results of evaluations for a series obtained from the experiment of boundary functions ($m = 2$ to 4) with a value for the Fourier number equal to 0.5 gave the magnitude of error as not exceeding 0.5%.

K. K. Vasilevskii
*Courtesy Referativnyi Zhurnal, USSR*

**3233. Glaser, H., Heat transfer in bulk goods** (in German), *Arch. Tech. Messen* no. 285, 199–202, Oct. 1959.

Author discusses heat transfer in process industries, such as occurs in regenerators containing heat-storage materials such as balls and small stones, Raschig-rings and other filling materials of metal or ceramics. If a flow of cold or hot gas takes place through such bulk material, then heat is transferred, the amount of which depends on the heat-transfer coefficient. Paper discusses the determination of this factor, for a given heat storer, by the principle of regenerator. In a regenerator, during a certain time hot gas passes through in one direction; then, in countercurrent procedure, during a similar time cold gas passes through in the other direction. This process is represented graphically in charts of temperature versus time, and of temperature along the length of the regenerator. Mathematical expressions are given for these processes. Author describes the experimental apparatus, shows layout and schematic section of the regenerator used. He illustrates and describes the platinum resistance thermometer used in such determination, the requirements for which are reliability and short response time; the variation of resistance is recorded through a Wheatstone bridge, by a string-galvanometer which has, likewise, a short response time. The results of measurement are represented in a chart, preferably in generalized form, as a function of the Nusselt number. A chart shows the Nusselt number as a function of the Reynolds number, for a Raschig-ring filling, from which the specific heat of the filling can be determined. Author discusses the correction necessary when the filling consists of large ceramic balls of laminar shape. He refers to numerous literature items for previous treatment of some pertinent and related problems.

K. J. DeJuhasz, USA

**3234. Ermakov, V. S., The nonstationary temperature field in heating elements of a reactor** (in Russian), *Inzhener.-Fiz. Zh.* **3**, 4, 127–131, Apr. 1960.

In designing the control system of a nuclear reactor one has to consider the distribution of the mean temperature along heat-producing elements in nonstationary conditions. Mathematically the problem reduces to solving the well-known equation of heat conduction in a finite rod with radiation at its surface and with both position- and time-dependent sources of heat. The value of the paper lies in a careful discussion of the rate at which heat is generated within the rod.

The solution is of different forms according to the type of reactor. It is simpler in the case of a reactor which is supercritical with regard to prompt neutrons, whereas the calculation with delayed neutrons leads to a rather complicated analytical formula.

V. Vodicka, Czechoslovakia

**3235. Lardner, T. J., and Pohle, F. V., A note on the application of the heat balance integral to problems of non-planar geometry,** AFOSR 57 (Polyt. Inst. Brooklyn, Dept. Aerospace Engng. Appl. Mech., PIBAL Rep. 585), 14 pp. + figs., Jan. 1961.

The temperature distribution in the region exterior to a cylindrical hole in the infinite space whose boundary is exposed to a constant heat flux is determined using the integral method with a parabolic profile. The results indicate that the use of the parabolic spatial temperature distribution leads to poor agreement with the exact solution. A modified profile is then introduced and is found to lead to better agreement. In addition, the analogous problem for the spherical cavity is treated.

From authors' summary

**3236. Baxter, A. N., Extension of the transient heating charts,** *ARS J.* **30**, 9, 904–905 (Tech. Notes), Sept. 1960.

**3237. Weissman, S., Saxena, S. C., and Mason, E. A., Intermolecular forces from diffusion and thermal diffusion measurements,** *Physics of Fluids* **3**, 4, 510–518, July/Aug. 1960.

Experimental values, previously obtained by two of the authors, for diffusion properties of the He-Ar, He-$CO_2$, and $H_2$-$CO_2$ are used in conjunction with the Chapman-Enskog theory to test various models of intermolecular forces. A consistency relation is derived to see if experimental data can fit *any* central force model. The results are that an exp-6 potential fits well for He-Ar, and Lennard-Jones fits fairly well for the other two systems. Combination rules for these two models are found to be satisfactory.

C. E. Pearson, USA

**3238. Oguri, T., On the coefficient of heat transfer between gases and cylinder walls of the spark-ignition engine,** *Bull. JSME* **3**, 11, 363–369, Aug. 1960.

An investigation was carried out on the heat transfer in a spark-ignition engine. Gas temperature in the cylinder was determined by computation from the pressure cards obtained by a Farnboro indicator. Periodic temperature changes on piston surface were also recorded. Analysis includes assumption of one-dimensional heat transfer through the gas layer and the wall together and constant wall temperature. Experimental results give an improvement of the Eichelberg's formula for heat transfer in engines by applying the law of similarity suggested by Elser. The result of this study, however, does not include generalization of the method of calculating the heat transfer for all engine speeds and engine dimensions.

S. L. Soo, USA

**3239. Oguri, T., Theory of heat transfer in the working gases of internal combustion engines,** *Bull. JSME* **3**, 11, 370–377, Aug. 1960.

This paper elaborates the theoretical aspects of the study reported in preceding review. It is an analytical study of the one-dimensional transient heat-transfer problem with heat sources in the gas phase, and density variation in the gas phase represented by polytropic processes. The effect of turbulence, as well as the effect of wall temperature variation, was neglected. The aim of this paper is to furnish a method for engine temperature calculations. Comparison with experimental results shows reasonable proximity of a theoretical model to the experimental system. Differences between calculated and experimental results were attributed to the method of accounting for heat generation. However, author's neglect of radiant heat transfer could contribute to a much greater factor.

S. L. Soo, USA

**3240. Perroud, P., and de la Harpe, A., Heat transfer by fluids propelled by a turbulent gas jet** (in French), *C. R. Acad. Sci. Paris* **249**, 23, 2503–2505, Dec. 1959.

**3241. Perroud, P., de la Harpe, A., and Weil, L., Heat transfer by liquids propelled by a turbulent gas jet** (in French), *C. R. Acad. Sci. Paris* **249**, 25, 2728–2730, Dec. 1959.

**3242. Brovkin, L. A., A method for the approximate calculation for the heating of bodies when the medium is subject to variations of temperature** (in Russian), *Izv. Vyssh. Uchebn. Zavedenii. Energetika* no. 6, 100–107, 1958; *Ref. Zh. Mekh.* no. 9, 1959, Rev. 10248.

An approximate method is put forward for the calculations of a nonstationary profile for the temperatures of the simplest one-dimensional bodies (an infinite plate, an infinite cylinder, a sphere) on the basic portion of the heating surface (Fourier number $\gtrless 0.3$), with the temperature of the medium varying with time. The main assumption made when using the method is that the curves showing the rise in temperature in the various points of the body shall be equidistant as regards time. The theoretical formulas are found in a simple form for a number of principles governing the changes of temperature of the medium. The numerical calculations by the given method agree quite well with the exact solutions. The accuracy of the method depends on the boundary conditions for the heating (with increase in the "Bio" numbers the precision improves).

K. K. Vasilevskii

*Courtesy Referativnyi Zhurnal, USSR*

**3243. Mizushina, T., Iuchi, S., Sasano, T., and Tamura, H., Thermal contact resistance between mercury and a metal surface,** *Inter. J. Heat Mass Transfer* **1**, 2/3, 139–146, Aug. 1960.

This paper describes a very carefully conducted experiment designed to determine the thermal contact resistance between liquid mercury and a solid surface. The results indicate a negligible resistance between pure mercury and chromium-plated copper.

While it is not the authors' intent to do so, they have also described a simple apparatus which can be used to measure directly the thermal conductivity of conducting materials.

J. A. Clark, USA

**3244. Murgai, M. P., and Emmons, H. W., Natural convection above fires,** *J. Fluid Mech.* **8**, 4, 611–624, Aug. 1960.

The equations for conservation of matter, energy and momentum in cylindrical coordinates are converted to total differential equations by considering that the same form of profile for the product (velocity × Δ specific weight)/(specific weight) occurs at each elevation. The effect of variable lapse rate is included by treating the lapse rate as piecewise continuous, and patching solutions together. Figures and working equations are given.

M. Tribus, USA

**3245. Asaturyan, A. S., and Tonkoshkurov, B. A., Free heat convection near a horizontal cylinder in highly viscous media** (in Russian), *Inzhener.-Fiz. Zh.* **3**, 6, 55–61, June 1960.

Authors use Pohlhausen method to obtain an approximate solution of the boundary-layer equations for a problem of practical interest. The boundary-layer thickness is assumed to be the same for temperature and velocity fields. Since small diameter cylinders are considered, an effective diameter is used to account for the thickness and the curvature of the boundary layer. The final equation is given by $Nu = 0.839(8/3 + Pr)^{-1/4}\, Pr^{1/4}(Pr \times Gr)^{1/4}$. The dependence of the Nusselt number on the factor $(Pr \times Gr)^{1/4}$ agrees with the results of earlier investigators; however, the coefficient is higher. Reviewer believes that the assumed velocity profile, $u = w(x)\,[1 - (y/\delta(x)^2]$, and the boundary condition, $\partial u/\partial y = 0$ at $y = 0$, are not realistic since the profile allows for the fluid slip at the surface of the cylinder. The choice of the velocity profile might account for some of the difference between the results of this paper and those of earlier investigators for the case of large Prandtl numbers.

Authors state that the correlation for Nusselt number was checked experimentally; however, the experiment is not described, and insufficient data are given in the table for it to be useful to other investigators. A correlation for Nusselt number based on experimental data for transmission oil and crude oil was obtained by the method of least squares and is given by $Nu = 0.882\,(Pr \times Gr)^{1/4}$. The diameters of wires used in the experiments were from 1.0 to 2.43 mm, and the range of $Pr$ and $Gr$ numbers covered were from 66 to 1300 and 0.3 to 3300, respectively.

R. Viskanta, USA

**3246. Goldstein, A. W., Stability of a horizontal fluid layer with unsteady heating from below and time-dependent body force,** NASA TR R-4, 15 pp., 1959.

The stability of a horizontal layer of fluid heated unsteadily from below and subjected to a time-dependent body force field was investigated theoretically, assuming an incompressible fluid with small density changes resulting from heating. A stability criterion is developed and used to calculate critical Rayleigh numbers, which are found to be much higher than for the static case, dependent only on the shape of the density profile, and independent of heating rate and Prandtl number. The initial motion corresponds to approximately the same cell shape as for the static case. Velocity growth increases from zero at the critical time at a rate proportional to the Prandtl number.

From author's summary

**3247. Newlander, R. A., Effect of shock impingement on the distribution of heat-transfer coefficients on a right circular cylinder at Mach numbers of 2.65, 3.51, and 4.44,** NASA TN D-642, 28 pp., Jan. 1961.

For this investigation a 2.8-inch-diameter cylinder was placed downstream of a $16\frac{3}{4}^\circ$ wedge on a flat-plate test surface which was mounted so as to produce boundary-layer thicknesses of 6 and 0.6 inches. The maximum heat-transfer coefficient on the stagnation line of the cylinder increased from $1\frac{1}{2}$ to 3 times the laminar theoretical value for a cylinder of infinite length at Mach numbers of 2.65 and 4.44, respectively. The maximum heating rate on the adjacent flat-plate surface immediately upstream of the cylinder increased from approximately 4 times the undisturbed flat-plate heat transfer at a Mach number of 2.65 to approximately 10 times the undisturbed flat-plate value at a Mach number of 4.44.

From author's summary

**3248. Warren, C. H. E., An experimental investigation of the effect of ejecting a coolant gas at the nose of a bluff body,** *J. Fluid Mech.* **8**, 3, 400–417, July 1960.

Nitrogen and helium coolant gases were ejected with and without a swirl from a bluff body nose in a 5.8-mesh stream. Pressure and temperature measurements were taken for incidences of 0, 4, 8° and heat flux calculated. Swirl ejection increased heat flux while straight-out ejection reduced heat flux provided the bow shock wave was not bulged out.

A. C. Mueller, USA

**3249. Chapman, S., Cosmic examples of heat conduction in very rare rotating or expanding gases** (in English), *Ann. Geophys.* **15**, 4, 434–444, Oct./Dec. 1959.

It was first shown by Maxwell that in very rare gases tangential stresses are produced by nonuniformity in the temperatures as well as by motion. In a similar way the rate of heat flow is affected and the usual law of heat flow has to be supplemented by additional terms. The theory of such phenomena is given in the book of Chapman and Cowling: "Mathematical theory of non-uniform gases." In the present paper cosmic applications are given for two types of motion: Rotation about an axis and radial expansion from a center. The gases considered have pressures of $10^{-6}$ atmospheres and consist of fully ionized and neutral atomic hydrogen as they occur in space. The additional terms in the rate of

heat flow may have some importance for cosmic physics, but are unlikely to affect the radial temperature distributions in the cases considered. H. Schuh, Sweden

**3250. Kokorev, D. T., Experimental methods applied to the determination of some temperature radiation parameters** (in English), *Inter. J. Heat Mass Transfer* **1**, 1, 23–27, June 1960.

Paper gives an analytical analysis of an experimental method for the determination of angle factors involving complex surfaces. The method requires measuring the equilibrium temperature of model surfaces as thermal energy is applied to the exterior of one surface. B. C. Watson, USA

**3251. Andriankin, E. M., Heat wave, radiating energy from front,** *Soviet Phys.-Tech. Phys.* **4**, 11, 1258–1262, May 1960. (Translation of *Zh. Tekh. Fiz., Akad. Nauk SSSR* **29**, 11, 1368–1372, Nov. 1959 by Amer. Inst. Phys., Inc., New York, N.Y.)

In this article, we investigate the propagation of a nonprogressize heat wave, radiating energy from the front. We examine the case for which the path of the radiation in the cold gas is large for all frequencies below the critical frequency $\omega_*$, and is small for high frequencies. The path of the quanta in the heated region is assumed to be much smaller than the radius of the wave front, since the radiant energy transfer takes place by means of heat conduction. From author's summary

**3252. Bevans, J. T., and Dunkle, R. V., Radiant interchange within an enclosure: Part 1, Absorption and emission behavior of gases; Part 2, General interchange equations; Part 3, A method for solving multinode networks and a comparison of the band energy and gray radiation approximations,** *ASME Trans.* **82** C (*J. Heat Transfer*), 1, 1–19, Feb. 1960.

Paper analyzes effect of contained gas on radiant interchange. Fundamental monochromatic equations are presented and used to develop equations for two approximations: band energy, and gray radiation. The latter is the basis of most engineering methods (e.g., Hottel, H.C., Chap. 4 of McAdam's "Heat transmission").

Authors present methods for both approximations, and illustrate their application to simple examples. The band energy approximation is more accurate, but requires more calculations and needs more knowledge of surface and gas properties. Reviewer considers this set of papers a significant achievement in the calculation of radiant transfer. R. R. Hughes, USA

**3253. Sparrow, E. M., and Albers, L. U., Apparent emissivity and heat transfer in a long cylindrical hole,** *ASME Trans.* **82** C (*J. Heat Transfer*), 3, 253–255 (Tech. Briefs), Aug. 1960.

Authors treat classical problem of a semi-infinite, evacuated, cylindrical cavity, having emissivity $\varepsilon$ and reflectivity $\rho = 1 - \varepsilon$. Accurate solutions of the integral equation for the apparent emissivity were obtained by numerical integration (mainly Simpson's rule) on a digital computer for values of $\varepsilon$ of 0.1, 0.25, 0.5, 0.75, and 0.9. Results are graphically presented. Comparison is made between these results and those presented by H. Buckley in 1927 and by E. R. G. Eckert in 1935, with the conclusion that the two schemes used by these investigators are quite useful. However, for greater accuracy, especially with low values of $\varepsilon$, the truncation error should be checked. J. W. Hlinka, USA

**3254. Wadsworth, P., Hot air drying—effects of temperature and humidity,** *J. Textile Inst. Proc.* **51**, 9, 552–562, Sept. 1960.

Paper gives a very elementary analysis of hot air drying of wet fabrics. The diffusion of water through the fabric from the interior to the surface is not dealt with. Eight figures illustrate the effects of temperature and humidity on the rate of drying and on the specific steam consumption. Author concludes that many old stenters are operating at an unnecessarily low level of efficiency and that many of them can be modernized to give reduced steam consumption and increased rate of drying.

G. Selin, Sweden

**3255. Imber, M., and Paschkis, V., Mathematical analysis of the rotary kiln heat exchanger: Part 1, The "well-mixed" condition** (in German and English), *Radex Rundschau* **4**, 183–197, Aug. 1960.

Heat exchange in a rotary kiln is a very complicated process, hard to analyze mathematically and almost equally difficult to study experimentally. Some attempts to tackle the problem experimentally are, however, now being made in Europe and this attempt to approach it mathematically is, therefore, timely and welcome.

A number of assumptions have to be made to make a solution feasible, and experimental confirmation is needed to see whether any major errors have thereby been introduced. Even so, the solution can only be carried out for two limiting conditions, conditions in an actual kiln being somewhere in between. The present paper gives solutions for the "well-mixed" case in which the temperature through the charge at any cross section is assumed constant and the heat transfer is higher than in practice. The other limit is when the charge is assumed to flow down the kiln as a solid slab.

The "well-mixed" case is worked out in detail and graphs for finding the optimum design of a kiln are presented and described.

This is a brave and important assault on an extremely complex problem and will be of interest to everyone dealing with rotary kilns. The continuation of this work and, especially, comparisons with experimental evidence will be watched for with great interest.

G. G. Thurlow, England

**3256. Malyshev, S. A., Experimental investigation of heat transfer and of hydraulic resistance of circulating draught electric resistance furnaces** (in Russian), *Trudi Mosk. Energ. In-ta* no. 30, 269–286, 1958; *Ref. Zh. Mekh.* no. 9, 1959, Rev. 10290.

Work was carried out on the subject given in the title; several types of bulk loads were tested on specially designed and constructed experimental plant. The coefficient of heat emission in a layer of fine fragments was determined on the basis of the theory of a regular regime. For charges consisting of metallic nuts type M16 and of the rings of ball bearings, relations were obtained which enabled calculations to be made for forced draught furnaces.

V. Ya. Chekhovskoi

*Courtesy Referativnyi Zhurnal, USSR*

**3257. Schalkwijk, W. F., A simplified regenerator theory,** *Trans. ASME* **81** A (*J. Engng. Power*), 2, 142–150, Apr. 1959.

The analysis resulted from a need for accurate values for the efficiency of regenerators. It was assumed that the temperature field in the central part of regenerators may be represented by simple linear or exponential functions. The development employed the concept of "enthalpy Flux," which makes it possible to calculate the loss of the regenerator from the zeroth eigenfunction. Hence it is possible to calculate the loss of a longer regenerator from the value for the loss of a shorter regenerator. The symmetrical regenerator in which the parameters have the same value for both flow paths is treated extensively. The results of the study of a general case are presented in the appendix. The formula which is derived is in agreement with published numerical results. G. A. Hawkins, USA

**3258. Krasnova, K. S., The calculating of the flow of heat and humidity by the nomographical method** (in Russian), *Uch. Zap. Kirovskoi Gos. Ped. In-ta* no. 15, 105–113, 1958; *Ref. Zh. Mekh.* no. 9, 1959, Rev. 10246.

The method is examined for constructing nomographs for determining flows of heat and humidity [see D. L. Laikhtman and A. F. Chudovskii, "The physics of the atmospheric layer next the earth," Gostekhizdat, 1949]. After carrying out some simpli-

fications and substitution of values for the calculation parameters the computational formulas are presented in the form

$$P = 0.298\,\varphi(\varepsilon, z_{00})\,u_1\,\Delta T \qquad [1]$$

$$E = 0.464\,\varphi(\varepsilon, z_{00})\,u_1\,\Delta l \qquad [2]$$

where

$$\varphi(\varepsilon, z_{00}) = \frac{\varepsilon^2 z_0^{2\varepsilon}}{(1-\varepsilon)^2 (z_1^{\varepsilon} - z_2^{\varepsilon})(z_2^{\varepsilon} - z_1^{\varepsilon})}$$

$$z_0 = z_1 \left[1 + \varepsilon \log_e \frac{z_{00}}{z_1}\right]^{1/\varepsilon}$$

Here $P$ is the flow of heat, $E$ the flow of humidity, $\varepsilon$ a dimensionless parameter, $\Delta T$ the difference in temperatures at heights of $z_1$ and $z_2$, $\Delta l$ the difference in elasticity of the water vapor at heights of $z_1$ and $z_2$, $z_{00}$ a variable magnitude, $z_1$ and $z_2$ constant magnitudes ($z_1 = 5$m, $z_2 = 20$m). A nomogram consisting of adjusted points and another nomogram in the form of an oriented transparency are constructed. From these, $P$ and $E$ may be obtained by the use of a ruler and the transparency once only.

G. S. Khovanskii
*Courtesy Referativnyi Zburnal, USSR*

**3259. Hughes, W. F., and Gaylord, E. W., On the theoretical analysis of a dynamic thermocouple: Part 2, The continuous area interface,** *ASME Trans.* **82** E (*J. Appl. Mech.*), 2, 259-262, June 1960.

Paper is a continuation of the earlier work reported by authors and their two other colleagues [*ASME Trans.* **80,** p. 307, 1958]. It considers the potential produced by a nonuniform Seeback emf distribution over a continuous circular area of contact between two semi-infinite bodies of dissimilar metal. Briefly, the method consists of finding an integral transform which relates the current density distribution function at interface, $\sigma(\vec{r})$, and the Seeback emf function $E(\vec{r}')$; $\vec{r}$ and $\vec{r}'$ are position vectors in interface. Following a procedure analogous to that used by Copson, authors developed equations from which the potential at infinity could be solved in terms of the spectral representation of the Seeback emf distribution function. The simple case with radial symmetry is treated in some detail and a numerical example is given. From the latter, authors conclude that the measured potential at infinity is not, in general, equal to area average Seeback emf. The temperatures near the perimeter of a contact area have greater influence on the measured potential than those in the interior. This finding is in qualitative agreement with authors' earlier paper (loc. cit) on randomly distributed source.

Reviewer believes that this paper and authors' earlier work provide valuable information to those who are concerned with the use of dynamic thermocouple to measure sliding contact temperature. However, he also feels that the value of the paper could be greatly enhanced if some numerical results were included and discussed for conditions actually prevailing at sliding contacts, where radial symmetry certainly does not exist. Such results are more relevant to what the paper tends to consider, namely, dynamic thermocouples. B. T. Chao, USA

**3260. Taylor, M. F., and Kirchgessner, T. A., Measurements of heat transfer and friction coefficients for helium flowing in a tube at surface temperatures up to 5900 °R,** NASA TN D-133, 29 pp., Oct. 1959.

Authors describe the apparatus and procedure used in determining fluid friction and heat transfer at high ratios of surface to fluid temperature, and correlate the data obtained. Average surface temperature was as high as 4533 °R, attained with an electrically heated tungsten or molybdenum tube. Physical properties of helium used in correlating data were calculated by published methods. Resulting correlations are of the usual kind and agree reasonably with those applicable at lower surface temperatures and temperature gradients.

Reviewer feels that because properties were calculated rather than experimental any conclusion regarding proper temperature to use in evaluating properties for correlation must be only provisional. Further, reviewer questions the significance of an average heat flux calculated from a temperature difference based on the average bulk fluid temperature when the fluid temperature rise is an appreciable portion of the difference between surface and fluid temperatures. Reviewer agrees wholeheartedly with authors in their statement "Since there could be some question as to the significance of average heat-transfer coefficients...... it seemed desirable to calculate local heat-transfer coefficients,"

C. E. Sanborn, USA

**3261. Schroder, J., A simple method of determining the thermal conductivity of solids,** *Philips Tech. Rev.* **21,** 12, 357-361, 1959/1960.

Author describes a unique experimental method and associated apparatus for determining thermal conductivities of solids. The two opposite faces of a cylindrical sample are maintained at constant temperature by the boiling of two different liquids having boiling temperatures which differ about 20 °F. Heat-transfer rate is obtained by measuring time to collect a given volume of condensate. Method is especially attractive if device is calibrated since $\Delta T$ is not affected by samples of different conductivity; only the time to collect a specific volume is changed. Calibration curve of thermal resistance against time to collect condensate is a straight line. Apparatus is constructed almost entirely of glass parts of which some are standard chemistry laboratory glassware. Problems of the method involve heat losses, temperature gradients in boiling liquid, determination of heats of vaporization, contact resistances, selection of sample dimensions, time to attain steady state, and selection of liquids. Compared with other methods these problems are generally less severe and especially so if the apparatus is calibrated. Paper represents a worthwhile contribution and is well written. A. J. Shine, USA

**3262. Kravchuk, E. M., Determination of the thermal coefficients of granular and solid materials by the method of plane thermal waves** (in Russian), *Inzbener.-Fiz. Zb.* **1,** 10, 29-37, 1958; *Ref. Zb. Mekb.* no. 9, 1959, Rev. 10260.

A description is given of a new method for the determination of the coefficients of thermoconductivity, temperature conductivity and thermal capacity of thermoinsulating materials by means of plane thermal waves in a body consisting of two substances.

From author's summary
*Courtesy Referativnyi Zburnal, USSR*

**3263. Suzuki, M., Theoretical and experimental studies on the vortex-tube,** *Inst. Phys. Chem. Res., Scient. Pap., Tokyo* **54,** 1, 43-87, Mar. 1960.

Vortex tube is a device which utilizes the centrifugal force produced by the vortex motion in a cylindrical tube. Flow of gases is three-dimensional, moving in the axial and radial directions with vortex motion. It is very difficult to carry out a theoretical analysis. Therefore, setting up some bold assumptions concerning the velocity profile, approximate analyses have been attempted from the aerodynamic, thermodynamic and acoustic standpoints. Results obtained are found to agree quantitatively with the experiments. From author's summary by M. S. Uberoi, USA

**Book—3264. Schack, A., Industrial heat transfer [Der industrielle Warmeubergang],** 5th improved and enlarged ed., Dusseldorf, Verlag Stahleisen M. B. H. 1957, xi + 434 pp. DM 43.75.

When this book first appeared in 1929, it was claimed in the preface that engineering applications were emphasized while re-

taining scientific rigor. Since then the book has been revised and enlarged and the latest (fifth) edition is now reviewed. While most books on heat transfer are written by university teachers, this one is particularly written for engineers by a member of the engineering profession; it differs appreciably from most other books on this subject. Little space is devoted to theory; boundary layers are mentioned only in a few lines, turbulence is hardly discussed at all, and only a few theoretical results are presented. Instead the book is based almost entirely on experimental results which are discussed at length. Heat-transfer coefficients are given separately for different media and expressed directly in metric units. Thus, the author points out, the reader is saved the trouble of calculating heat-transfer coefficients from nondimensional expressions. Different experiments are not correlated with the help of nondimensional quantities, but separate correlations are made for the dependence on speed, diameter and so on. This is an obvious consequence of the author's way of presenting his material, but can not be considered as a good basis for comparison.

Many modern results on fundamentals of heat transfer are missing in the book; among them are correlations which are now generally accepted as standard, such as those of Hilbert for forced convection on cylinders and those of McAdams for free convection on horizontal cylinders and vertical flat plates. Further there is no information on the local distribution of the heat-transfer coefficient on bodies. On the other hand, heat transfer in the entrance region of tubes is treated in detail. In the chapter on thermal radiation between walls only the fundamental laws are given, but heat exchange between surfaces of different configurations is not further dealt with. Thermal radiation of gases is smply treated. A considerable part of the book is devoted to technical applications: Heat exchangers in steady and unsteady flow, heat transfer in burners, pressure drop in tubes and bundles of tubes, and the relation between heat transfer and pressure drop. In these parts the engineer will find much valuable material. A large section is devoted to numerical examples and extensive tables.

The book is easy to read and would certainly appeal to engineers who want results quickly. In the field of "conventional" engineering the book obviously serves by and large its purpose, but readers must be cautioned against omissions of important material. The book would in general not be very useful for those dealing with more advanced problems.

H. Schuh, Sweden

## Combustion

**(See also Revs. 3019, 3238, 3244, 3252, 3271, 3274, 3305)**

**3265. Hirschfelder, J. O., Curtiss, C. F., and Barnett, M. P., Ignition temperature approximation for bimolecular detonations,** Univ. Wisconsin, Theoretical Chem. Lab. CM-976, Series 8, 6 pp., July 1960.

This article presents a quantitative theoretical chemical study of bimolecular detonations. Gaseous detonations supported by the bimolecular reaction $2A \rightarrow 2B$ are studied, assuming that an ignition temperature approximation may be made. Detailed numerical calculations are made of thermodynamic and transport properties of the gas undergoing the detonation. From this it is found that there is little effect of chemical reaction order on the properties of detonation. Also, it is found that for some values of the parameters considered, the von Neumann pattern of the shock zone and chemical reaction zone decoupled, is very nearly approached. However, Hirschfelder, Curtiss and Barnett find that "because of the unrealistic nature of ignition temperature chemical kinetics, it is not possible to reach any conclusions regarding experimental gaseous detonations."

The article is a continuation of previous work by Hirschfelder et al. [Curtiss, Hirschfelder, Barnett, *J. Chem. Phys.* **30**, p. 470, 1959, and Hirschfelder, *Physics of Fluids* **3**, 2, 210–216, 1960; AMR **13**(1960), Rev. 6000] and is written on a plane most suitable for the physicist or theoretical chemist.

I. J. Eberstein, USA

**3266. Williams, F. A., Penner, S. S., Gill, G., and Eckel, E. F., Heterogeneous burning in a diverging reactor,** *Combustion and Flame* **3**, 3, 355–371, Sept. 1959.

Authors develop a computational model of heterogeneous combustion involving either two liquid reactants or one liquid and one gaseous reactant; all products are gaseous. Model assumes no slip between phases, one-dimensional flow, and a high volume ratio of gas to liquid. Two different spray distributions are considered: uniform size, and a Rosin-Rammler distribution. Predicted curves of combustion rate versus distance along the reactor are compared with values calculated from experimental pressure profiles. To obtain approximate agreement, small drop sizes (ca 7 microns) must be postulated.

Reviewer finds comparison of experimental and predicted results unsatisfactory. Conversion of pressure-profile data to compare with predicted reaction rates requires extensive calculation. Yet curve obtained is of hyperbolic shape, making the graphical comparison difficult. Direct comparison of predicted and experimental pressure profiles would be most desirable; if calculated values must be used, they should be chosen to clarify the graphical comparison. For future studies, reviewer recommends using the upper-limit distribution [Mugele and Evans, *Indust. Engng. Chem.* **43**, p. 1317, 1951] rather than Rosin-Rammler, since the latter can result in incorrect mean diameters.

R. R. Hughes, USA

**3267. Vandenkerckhove, J. A., Influence of erosive burning on internal ballistics of solid propellant rockets,** AFOSR TN 60-1403 (Univ. Brussels, Inst. Aero. TN 1), 49 pp., Oct. 1960.

Reviewer believes this paper is an important addition to the literature on solid propellant rockets. As summarized by the author "... Empirical laws do not describe accurately the influence of erosive burning and the factors influencing the burning rate without erosion apparently do not retain their influence in the pressence of a high velocity flow. It is more preferable to base internal ballistics (gas flow) computation on actual experimental data and a method is proposed for calculating the front pressure as a function of combustion-to-throat, and throat-to-port area ratios (burning surface/throat area and throat area/propellant port area). The method is rigorous for a constant port area and it is also shown that a small port-area taper along the grain length does not introduce a significant error. Finally, the problems of constant mass velocity and constant burning rate designs are briefly discussed...."

Additions in parentheses are the reviewer's.

A. S. Andes, USA

**3268. Priem, R. J., and Heidmann, M. F., Propellant vaporization as a design criterion for rocket-engine combustion chambers,** NASA TR R-67, 127 pp., 1960.

Calculated vaporization rates and histories for sprays of heptane, ammonia, hydrazine, oxygen, and fluorine are presented. The results are correlated with an effective chamber length for ease in using them for design purposes. Vaporization rate can be increased by decreasing drop size, initial velocity, and contraction ratio or by increasing chamber and nozzle length, pressure, and initial propellant temperature. With a method of analysis that relates combustor performance to the quantity of liquid propellant vaporized, experimental and calculated results are compared to show that incomplete vaporization of a small number of large drops is responsible for a loss in engine performance.

From authors' summary

**3269. Spengler, G., and Gemperlein, H., Small combustion chamber for the rapid testing of jet engine fuels** (in German), Deutsche Versuchsanstalt Luftfahrt, Ber. 80, 100 pp., Dec. 1959.

Authors describe a subsonic ramjet which has been developed for the comparative study of fuels used in jet engines. The test-stand is applicable to fuels intended for engines with moderate pressure ratios only, in which the temperatures of the air in the combustion chamber are too low for self-ignition. The test-stand was designed with newly developed fuels in mind, and while it was important to simulate the steady flow combustion of real engines as far as possible, fuel economy had also to be considered to enable the testing of new fuels which were available in small quantities only.

The theoretical portion of the report is devoted mainly to the derivation of gas dynamics relations for the ramjet, some of which are well known. The main interest of the experimental results lies in the measurement of optimum thrust with various fuel mixtures. Tests were carried out with various alcohol-benzol mixtures having varying calorific values but nearly constant boiling points, and with benzol-tetralin mixtures having varying boiling points and only slightly varying calorific values. The experimental results are still tentative in character, and authors intend to report on further results. Y. R. Mayhew, England

**3270. Kogarko, S. M., Amplification of compression waves in interaction with a flame front,** *Soviet Phys.-Tech. Phys.* **5,** 1, 100–108, July 1960. (Translation of *Zh. Tekh. Fiz., Akad. Nauk SSSR* **30,** 1, 110–120, Jan. 1960 by Amer. Inst. Phys., Inc., New York, N. Y.

Benzene-air and aviation gasoline-air mixtures were burned in a spherical combustion vessel over a range of pressures and fuel-air ratios. The flame was photographed with a drum camera and pressure measurements were made with a differential pressure indicator. Studies indicated that under certain conditions the compression wave generated by an accelerating flame front is amplified. Author suggests that when compression wave enters flame reaction zone incomplete statistical equilibrium results and an increase in temperature and pressure occurs. This action develops pressure perturbation which is propagated in direction of compression wave front and causes it to be amplified. Amplification factor is found to be a maximum for high chemical reaction rate, thus is a function of mixture composition. Experimental results were verified by creating artificially produced compression waves with small charges of lead nitride. These waves were amplified if they exceeded some critical value of intensity.

T. P. Clark, USA

## Prime Movers and Propulsion Devices

(See also Revs. 2851, 2858, 3238, 3266, 3267, 3305)

**3271. McLafferty, G. H., Relative thermodynamic efficiency of supersonic combustion and subsonic combustion hypersonic ramjets,** *ARS J.* **30,** 11, 1019–1021, Nov. 1960.

Comparison is made between relative merits of supersonic and subsonic combustion in hypersonic ramjets under identical free-stream conditions and for equal equivalence ratios and total pressure losses in supersonic diffusion. Subsonic combustion model considered is unconventional in that transition from supersonic to subsonic flow is affected through single normal shock within constant-area passage in which combustion takes place; and Mach number of this shock is stipulated to be equal to inlet Mach number of supersonic combustor. Under these special conditions, supersonic combustion is found to become preferable to subsonic combustion at Mach numbers above approximately 7 for ramjet with external-compression inlets. J. V. Foa, USA

**3272. Nason, M. L., Terminal guidance technique for satellite interception utilizing a constant thrust rocket motor,** *ARS J.* **30,** 9, 902–905 (Tech. Notes), Sept. 1960.

The planar relative motion between a target and interceptor satellite is treated by perturbation techniques to obtain linearized equations of motion which are valid in the vicinity of a collision course. A terminal guidance law is formulated for attaining a collision course and for the achievement of satellite rendezvous using constant thrust rocket motors.

From author's summary

**3273. Brocher, E., Study of rocket propelled with hydrogen** (in French), *Bull. Tech. Suisse Rom.* **84,** 9, 161–166, Apr. 1958.

This is a study made at the Institut fur Aerodynamik, ETH (= Eidgenossische Technische Hochschule = Ecole polytechnique federale), Zurich, under the direction of Professor J. Ackeret. It gives a very nice outline of the basic interior ballistic limitations of a rocket in which the ejected material can be heated by some nuclear reactor. For high velocities it is necessary to have a large ratio of $c_p/M$, and hydrogen easily wins over helium; for not only is there the gain in $1/M$ and the gain in $c_p = R\gamma/(\gamma - 1)$ as reckoned for ideal gas, but also a radical increase in $c_p$ is furnished by the dissociation of the hydrogen, particularly at low pressures and high temperatures. There are graphs (unfortunately, very much reduced in scale) of the degree of disassociation $\alpha$ and of $c_p$ as functions of $p$ and $T$; the $T$-$S$ chart for hydrogen; and graphs of specific impulse for helium and hydrogen, as function of temperature and nozzle expansion ratio. Without dissociation (up to some 3000°K) hydrogen yields effective velocities some 1.5 times higher than helium; while at 5000°K effective velocities are roughly 15,000 vs 6,000 m/s. Brocher points out the existence of effects of nozzle size and nozzle expansion angle upon relaxation, but limits his calculations (introducing weighted $c_p$ and $\gamma$) to the case when recombination is instantaneous. That the effect of relaxation cannot be very great is shown by an example with $T_0$ = 5000°K, expansion ratio 1/0.0047. With strong dissociation ($p_0$ = 5 bar, $\alpha$ = 0.832) physical exit velocity is 13,160 m/s; with moderate dissociation ($p_0$ = 80 bar, $\alpha$ = 0.350), it drops to only 12,330 m/s (finite relaxation would reduce these figures, particularly the greater one). Noting that one limitation on velocity is the loss of energy of dissociation in the jet, author also computes exit velocity (for $p_0$ = 5 bar) with complete expansion. For 4500°K, this is nearly 21,000 m/s. Reviewer believes that label $c_{1\,max}$ on right-hand scale of Fig. 4 should be removed, and that figure 1.09 on Fig. 7 ought to be 1.45. S. J. Zaroodny, USA

**3274. Ciepluch, C. C., Temperature histories in ceramic-insulated heat-sink nozzle.** NASA TN D-300, 25 pp., July 1960.

Author investigates the temperatures of composite nozzle walls of a type used for solid propellants of high performance and long burning times. The effective flame temperature was calculated according to Huff et al. [NACA Rep. 1037] and Gordon et al. [NASA TN D-132]. The heat-transfer coefficient in the nozzle was taken as that for a tube of constant cross section. Heat conduction in the nozzle walls was calculated by the finite difference method. Optimum values for the wall thicknesses were found for given temperature limits of the materials, given operating conditions and required running times. The measured temperature increases were less than those calculated and this is explained by the simplifications in the following assumptions: (a) The influence on heat transfer of the inhibitor and of the case insulation of the propellant had been neglected and (b) in calculating the heat flow in the nozzle the annular wall had been replaced by a slab.

H. Schuh, Sweden

**3275. Matthews, C. W., and Cuddihy, W. F., Experimental study of a single-coil induced-electromotive-force plasma accelerator,** NASA TN D-639, 24 pp., Jan. 1961.

An experimental study was made of a single-coil induced-electromotive-force plasma accelerator which used a capacitor discharge for the driving force. A strong shock was observed from the first pulse with a velocity of $10^6$ centimeters per second. This shock was followed by three or four discharges which produced plasmoids moving at about $5 \times 10^6$ centimeters per second. The efficiency of the accelerator was estimated to be about 3 per cent in the production of the high-velocity plasmoids. Suggestions are made for the improvement of this type of accelerator.

From authors' summary

## Magneto-fluid-dynamics

(See also Rev. 3316)

**3276. Chopra, K. P., Excitation of plasma waves by bodies moving in ionized atmosphere,** AFOSR 1 (Polyt. Inst. Brooklyn, Dept. Aerospace Engng. Appl. Mech., PIBAL Rep. 627), 11 pp., Jan. 1961.

A body moving in an ionized atmosphere acquires an electric charge through the processes of accretion of charged particles and emission of electrons by high-energy photons. The moving charged body may then interact with the charged particles of the atmosphere and any pervading magnetic field to excite plasma waves. Of particular interest is the situation in which the body collects an ionized cloud in front of it. The motion of this ionized cloud in the atmosphere induces an electrostatic instability and causes a column of ionized gas to move ahead of the body. The electrostatic instability is conducive to the excitation of electrostatic oscillations which, if already present, are further enhanced. A magnetic field along the direction of motion assists in the formation of the ionized cloud. If the pervading magnetic field is of suitable weak strength, it may excite extraordinary electromagnetic waves. A pervading transverse magnetic field of suitable strength may cause the excitation of magnetohydrodynamic waves.

From author's summary

**3277. Williams, W. E., Propagation of small amplitude magnetohydrodynamic waves,** *Quart. J. Mech. Appl. Math.* **13**, 3, 272-277, Aug. 1960.

Author considers linearized equations of motion of a viscous compressible fluid from the viewpoint of generating solutions in terms of one component vector potential. The starting point is linearized equations of magnetohydrodynamics (hydrodynamic plus Maxwell), i.e., there are deviations of the pressure and density from the uniform state. It is possible to show that the scalars $p$ and div $\vec{E}$ satisfy independent homogeneous differential equations. Hence it seems reasonable to seek solutions of the system in question with either $p$ or div $\vec{E}$ = zero; and hence divide the general solution into two classes generated by assuming either of these conditions.

Author considers first waves with zero excess pressure (the so-called $\nu$-modes). This reduces the system to a single equation with possible two plane wave solutions. The class of solutions generated by div $E = 0$ (called the class of solution pressure modes) refers to a single equation obtained from the system in question by simple operations. Hence the original system of nine first-order equations has been reduced to two equations: fourth and fifth-order.

The obtained solutions are particularly useful for investigating time harmonic cylindrical waves and plane waves. It is shown that results obtained by Ludford [*J. Fluid Mech.* **5**, p. 387, 1959] represent only a particular case of the present results, i.e., when the coefficient of viscosity is equal to zero. As a particular case the author considers the standing waves in a rectangular box (analogous to that of Ludford).

M. Z. v. Krzywoblocki, USA

**3278. McCune, J. E., and Resler, E. L., Jr., Compressibility effects in magnetoaerodynamic flows past thin bodies,** *J. Aero/Space Sci.* **27**, 7, 493-503, July 1960.

Authors have presented a competent study of a compressible conductor flow past thin cylindrical bodies, essentially in the region of a very large (magnetic) Reynolds number, over the entire Mach number-Alfvén number range. The elliptic, hyperbolic and hyper-liptic (superposition of an elliptic and a hyperbolic field) steady flow regimes were investigated in relation to aligned, as well as crossed fields, for which some detailed solutions were worked out. The feasibility of treating these problems by analogies with the Ackeret theories (conventional aerodynamic), and by the propagation of magnetoacoustic pulses, appeared well established.

The derivations include small-perturbation equations, followed by the two basic orientations of magnetic field, namely the parallel and the perpendicular one ("aligned and crossed") to the free stream. Extensive mathematics (89 equations) is featured and eight informative graphs assist in illustrating the flow problems, the stipulated magnetoaerodynamic cases and a graph of the "modified Mach lines."

The physical interpretation of the derived theories represents a fine contribution to the knowledge of applying magnetoaerodynamic phenomena to the contemporary advanced design concepts of supersonic flight and the assessment of criteria in its related problems.

The research work is documented by eleven (11) references covering some of the principal investigations previously published on the subjects of magnetoaerodynamics, magnetohydrodynamics and magnetoacoustics.

C. R. Bell, USA

**3279. Williams, W. E., Propagation of electromagnetic surface waves along wedge surfaces,** *Quart. J. Mech. Appl. Math.* **13**, 3, 278-284, Aug. 1960.

Author investigates the excitation of surface waves by a plane electromagnetic wave incident on a wedge. A cylindrical polar coordinate system is chosen with its axis along the vertex of the wedge. The mathematical problem begins with the solution of the equation $(\nabla^2 + k^2)\phi = 0$ subject to the appropriate boundary conditions. It is also assumed that a plane wave is incident on the wedge. The solution is obtained in terms of exponential functions and it is split up into two parts, which is particularly suitable for small values of the impedance. Next, the author discusses thoroughly the properties of the solutions, in particular the poles; the integrand appearing in the solution is required to have no poles in a certain strip; hence, another function of the solution may be required to have zeros which cancel these poles. The behavior at infinity of the function representing the solution is discussed in the last part of the paper.

With the poles segregated into two classes, the first class of poles is seen to give rise to a term which behaves like the exponential function; this is of the surface-wave type. Thus it is seen that there is a possibility of obtaining surface-wave terms in the present problem. The second class of poles clearly produce surface waves on the surface. For simplicity the author considers only the first class of poles; this corresponds to investigating the surface waves on $\theta = 0$. The author constructs a method by means of which the pole producing the surface-wave term is captured. But the result obtained by the author does not agree with the corresponding result of F. C. Karal and S. N. Karp [New York Univ. Res. Rep. E. M123, 1959].

M. Z. v. Krzywoblocki, USA

**3280. Kendall, P. C., Hydromagnetic oscillations of a rotating liquid sphere,** *Quart. J. Mech. Appl. Math.* **13**, 3, 285-299, Aug. 1960.

Author considers a liquid sphere rotating with uniform angular velocity in a uniform magnetic field parallel to the axis of rotation of the sphere; the angular velocity is small so that we may neglect the departure of the surface from the spherical form. The conductivity is assumed to be infinite, the lines of force are frozen in the fluid. The starting point is the linearized equations of motion in cylindrical polar coordinates. The equation of continuity of the liquid and the equation div $\vec{b} = 0$ ($\vec{b}$ magnetic field intensity vector) are satisfied by introducing the hydrodynamic and magnetic stream functions. Simple operations lead to the final partial differential equation in the small variation of the angular velocity. A solution to this equation is proposed in terms of the Bessel functions of order 1, and next in the form of exponential series, whose constants are the roots of the characteristic equation.

The next problems considered are those of the boundary conditions, the pressure equation associated with an infinite set of equations involving the spherical Bessel functions, the magnetic field variations, the effect of gravitational forces on the oscillations of a liquid globe, the separation of the oscillations into even and odd modes, and successive approximations to the fundamental periods. The convergence of the above solutions is not very rapid. It turns out that the exponential terms only take effect toward the poles of the sphere. Near the equator the convergence series is better. M. Z. v. Krzywoblocki, USA

**3281. Ring, L. E., Unsteady magnetohydrodynamic flow about thin airfoils,** AFOSR TN 60-638 (Cornell Univ., Grad. School Aero. Engng. Rep.), 143 pp., June 1960.

The theory is developed for the magnetohydrodynamic flow of an incompressible fluid about thin airfoils in nonuniform motion. A uniform magnetic field is applied parallel to the free stream and solutions are obtained subject to the restriction of small perturbations. The effects of viscosity are included, for the most part, only through the application of the usual Kutta condition for lifting airfoils. The general character of the flow is discussed at length. The validity and range of applicability of the infinite-conductivity and moderate-conductivity theories are determined on the basis of an order-of-magnitude analysis.

The flow-field for infinite conductivity is changed from the nonmagnetic case only through the new transport speed of vorticity; the forces on the airfoil are changed due to surface currents. For the case of the Alfvén speed less than the free-stream speed, the airfoil lift and pitching moment are given in integral form for general unsteady-airfoil motion and are given in closed form for harmonic oscillations. The forces at moderate frequencies are found to be larger than the corresponding nonmagnetic case. The response to a unit step-function change in the downwash is studied and the asymptotic form of the lift is obtained for small and large time.

Using the Oseen approximation, a set of singular solutions with arbitrary conductivity and magnetic field strength are obtained for harmonic oscillations. Limiting forms of these singular solutions are given for low conductivity and for low frequencies. Using these solutions, integral equations are formulated for the case of an oscillating airfoil, which may be a conductor or an insulator.

From author's summary

**3282. Shmoys, J., and Mishkin, E., On the linear behavior of large-amplitude magnetohydrodynamic waves,** *Physics of Fluids* **3**, 4, 661-662 (Letters to the Editor), July/Aug. 1960.

**3283. Peyret, R., On a similarity between magnetodynamic liquid flow and dynamic flow of gases** (in French), *C. R. Acad. Sci. Paris* **250**, 11, 1971-1973, Mar. 1960.

# Aeroelasticity

(See also Revs. 2981, 3161, 3164)

**3284. Walberg, G. D., Subsonic kernel-function flutter analysis of a highly tapered tail surface and comparison with experimental results,** NASA TN D-379, 37 pp., Sept. 1960.

A flutter analysis employing the kernel function for three-dimensional, subsonic, compressible flow is applied to a flutter-tested tail surface which has an aspect ratio of 3.5, a taper ratio of 0.15, and a leading-edge sweep of 30°. Theoretical and experimental results agree reasonably at Mach numbers from 0.75 to 0.92. At Mach numbers from 0.92 to 0.98, however, a second solution to the flutter determinant results in a spurious theoretical flutter boundary at a much lower dynamic pressure and at a much higher frequency than the experimental boundary.

From author's summary by M. Botman, USA

**3285. Kapzon, G. I., Vibration of a thin rectangular wing of large aspect ratio in a supersonic stream,** *Appl. Math. Mech.* (*Prikl. Mat. Mekh.* ) **22**, 6, 1153-1161, 1958. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

Aeroelastic stability and dynamic response of a uniform, finite beam-rod exposed to a supersonic stream of perfect gas are analyzed, representing the deformation as a Fourier series in the spanwise coordinate. Aerodynamic lifts and moments are calculated from unsteady linearized theory which accounts for three-dimensional effects due to spanwise angle-of-attack variations but neglects the tips. Starting from beam-rod differential equations and Laplace-transforming on time, it is found that each Fourier component is uncoupled and is described by a pair of algebraic equations in the *n*th bending and torsional amplitudes. The coefficients contain transcendental functions of the Laplace variable which have been tabulated for purely imaginary arguments.

Although no explicit solutions are given, the problem of flutter in the *n*th mode, related to the determinant of the *n*th system of equations, is examined in detail. Determination of the characteristic roots is simplified for the case when the ratio of flight dynamic pressure to torsional stiffness is small. Rapidly-convergent series representations of the aerodynamic coefficients are developed. The question of structural boundary conditions associated with the chosen mode shapes is not discussed.

H. Ashley, USA

**3286. Davidson, J. R., Optimum design of insulated tension members subjected to aerodynamic heating,** NASA TN D-117, 38 pp., Dec. 1959.

Analysis considers heat to be stored in metal member as sink, so that time of flight is important variable. "Structural efficiency" is based on minimization of total weight of member and heat insulation. By using known material properties, load parameter is derived which makes possible construction of efficiency curves for various materials without specifying insulation and flight time. Results indicate beryllium is excellent material. Aside from Be, Ti is most efficient for short flight time and high load; stainless steel and Inconel X become most efficient as flight time increases and load decreases. Results are based on simplified one-dimensional case, with various assumptions, which may raise questions as to applicability to more complicated actual structures. However, paper is good contribution, is relatively brief and easy to follow, and should help in preliminary design studies.

C. W. Smith, USA

**3287. Broglio, L., The equielastic model technique in blowndown wind-tunnel tests,** AFOSR 59-409 (Universita di Roma, Scu. Ingegneria Aero. SIAR. 46, TN-6) ASTIA AD 214506), 44 pp., Dec. 1958.

Paper is one publication in a series of Technical Notes dealing with solution of aeroelastic problems in supersonic blow-down wind tunnels. For an easier understanding of the paper, the previous publications of the series should be on hand. The basic concepts used in the paper are the "Balance Method" and the "Equielastic Model," introduced by author in previous publications. Paper gives a general procedure with numerical examples for determining the equielastic model, consisting of the two main problems: (a) Find pressure distribution on a solid model when the spanwise and chordwise rotations are known. (b) Determine the tip rotation for a given wing structure. Aeroelastic coefficients obtained in the computations show the relatively small effects of Reynolds number compared with the effect of stagnation pressure over modulus of elasticity. H. J. Ramm, USA

**3288. D'Ewart, B. B., and Farrell, R. F., Details of a new approach to safe flight flutter testing,** AFOSR TN 60-1476 (Bell Aerosystems Co. Rep. 7-60-918002), 104 pp., Nov. 1960.

## Aeronautics

(See Revs. 3093, 3140, 3154, 3164, 3169, 3173, 3214, 3271, 3278)

## Astronautics

(See also Revs. 2847, 2850, 3167, 3355)

**3289. Garofalo, A. M., New set of variables for astronomical problems,** *Astronomical J.* **65**, 3, 117-121, Apr. 1960.

Author considers the problem of the motion of a particle in the field of $n$ other particles with non-gravitational forces permitted. In a Keplerian orbit when the commonly used elements are employed, singularities or indeterminacies occur when the inclination vanishes or when the orbit is either circular or parabolic. An orthogonal transformation of the position and velocity vectors removes these in each special case. Identifying three of the new variables, the author derives differential equations, in the general $n$ body case, for these as well as for the matrix elements of the transformation, giving the user some freedom of choice for the three remaining variables in order to avoid singularities while preserving, as desired, their constancy in the pure Keplerian case. In spite of the general treatment it is not demonstrated that there is any computational advantage over commonly used variables in nonsingular situations. In the nearly singular cases there are already a number of ad hoc orthogonalizations in practice.

The effort of an average reader to follow the derivation will be eased by the initial realization that the summation convention of tensor analysis is used and that a couple of unfortunate but recognizable typographical errors intrude.

M. L. Juncosa, USA

**3290. Ting, L., Optimum orbital transfer by high thrust rockets,** AFSOR 163 (Polyt. Inst. Brooklyn, Dept. Aerospace Engng. Appl. Mech., PIBAL Rep. 633), 29 pp. + figs., Feb. 1961.

For a high thrust rocket, the specific impulse is of the order of or larger than the local circular orbital velocity, and the fuel consumption rate is so large that the dimensionless parameter, $\varepsilon$, defined as the ratio of the product of initial weight and specific impulse to that of the fuel consumption rate and a reference radial distance to the earth's center or the common focus, is much less than unity. The solution of transfer by a single impulse is shown to be equivalent to the true optimum transfer by a high thrust rocket when terms of the order of $\varepsilon$ are neglected.

With the aid of the approximate theory for variational problems, developed in a previous paper, the solution of transfer by a single impulse is improved so that the result will differ from the true optimum by the order of $\varepsilon^2$ only. Explicit formulas for the improvements in the total burning time and in the thrust angle program are obtained. With application of these formulas to each impulse transfer, it is shown that an optimum solution of transfer by $N$-impulse can be improved so that the result will differ from the true optimum transfer by $N$ high thrust rockets by an order of $\varepsilon^2_{max}$ where $\varepsilon_{max}$ is the largest of the $N$ small parameters $\varepsilon_j$ associated with each rocket. From author's summary

**3291. Anthony, M. L., and Fosdick, G. E., Escape in the equatorial plane of an oblate planet,** *ARS J.* **30**, 9, 898-901 (Tech. Notes), Sept. 1960.

This note investigates the influence of oblateness on the escape path of a vehicle moving in the equatorial plane of a planet, and presents some numerical results pertinent to the case of Earth.

From authors' summary

**3292. Vertregt, M., Graphical plotting of interplanetary orbits,** *J. Brit. Interplanetary Soc.* **17**, 10, 351-358, July/Aug. 1960.

A simple, ingeneous, geometric construction is described by means of which a point on a conic section can be found if either the radius or the true anomaly are given and two parameters are known. Geometric methods are given of constructing tangent lines to conic sections, graphical construction of time of transfer from one point to another along an elliptic orbit and of the direction and magnitude of velocity needed to make an impulsive transfer from one orbit to another. All of the latter methods apply only to a central force field without perturbations.

R. E. Street, USA

**3293. Lawden, D. F., Necessary conditions for optimal rocket trajectories,** *Quart. J. Mech. Appl. Math.* **12**, 4, 476-487, Nov. 1959.

It is known that the problem of transporting a rocket through a particular gravitational field from a given initial state to a desired terminal state, with minimal expenditure of fuel, can be viewed as a special case of Mayer's problem, a general problem in the calculus of variations with a highly-developed theory. In this well-written paper the author uses this knowledge to show, among other things, that optimal trajectories are composed of maximum and zero-thrust segments (no aerodynamic forces are present). As illustrations, treatments of the cases of unbounded thrust and motion in a uniform gravitational field are sketched. No computational considerations are included. R. Kalaba, USA

**3294. Weber, R. J., and Pauson, W. M., Some thrust and trajectory considerations for lunar landings,** NASA TN D-134, 37 pp., Nov. 1959.

Paper studies lunar landing pattern composed of retrorocket change from circumlunar orbit to elliptic lunar orbit to retrorocket change to lunar vertical descent to retrorocket soft landing and limited variations from this pattern. The study examines sensitivities of the thrust-weight ratio and speed-change parameters and the significance of timing and position errors. It is shown for example that speed change is very critical in the first step whereas thrust alignment is much less important. Related equations are developed in an appendix and illustrative graphs are presented.

M. G. Scherberg, USA

**3295. Fridlender, G. O., System for determination of the parameters of motion of a body in space** (in Russian), *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk* no. 6, 108-117, Nov./Dec. 1959.

It is known that the errors of the attractive field and the instrumental defects of elements of a system (accelerometers and inte-

grators) moving in space can be great. The correction used in analogous geosystems because of "nonweights" cannot be used. Hence author uses two optical systems directed toward the sun and a planet. As an example the spherical coordinate system is used with the origin in sun, the initial plane of which coincides with the ecliptic plane and the initial meridian passes through the point of weight equilibrium. This system is very near to the orthodromic one. The position of the first platform is determined by means of two angles referred to the three gyroscopic force system. The period of vibration of the platform and the corresponding coefficient of the motion are determined assuming an accuracy of 0.5% of the velocity of the motion of apparatus, semi-aperiodic damping, and the amplitude of the vibration of platform less than 10 to 12 in.

D. Raskovic, Yugoslavia

**3296. El'yasberg, P. E., Dependence of secular variations of orbit elements on the air resistance,** NASA TT F-47, 10 pp., Nov. 1960.

To obtain approximate expressions for the secular variations of the orbital elements due to air resistance, the perturbative functions are expanded in power series with the use of Bessel functions. These series are, however, not truncated but resubstituted by equivalent compact expressions yielding not only handy but quite accurate expressions for the variations of the: semi-latus rectum, eccentricity, semi-major axis, period and perigee radius due to drag during one orbital revolution.

Comparing with results obtained by numerical integration, it was found that for eccentricities greater than 0.04 but less than 1 the error was of the order $1 \sim 2\%$. For smaller eccentricities the error increases; however, the case of zero eccentricity can be again treated quite accurately by another set of included formulas.

G. S. Gedeon, USA

**3297. Proskurin, V. F., and Batrakov, Yu. V., First-order perturbations in the motion of earth satellites due to oblateness of the earth,** NASA TT F-45, 12 pp., Nov. 1960.

This is the first version of a paper published by the authors in 1959 on the subject of first-order perturbations of satellites due to oblateness effects. The classical equations of Lagrange are expanded in powers of the eccentricity including the third power.

In 1960 the authors published an improved version of the same paper which was translated in the 1961 January issue of the *ARS Journal*. The expansion in this later version contains the eccentricity terms including the fifth power. Additionally, the second-order secular perturbations of the node are treated. With the derived formulas and with observation of the regression of the nodes, the oblateness of the Earth can be determined with accuracy.

The theory presented in these papers is not new in this country, see e.g., S. Herrick: "Astrodynamics," Van Nostrand Co.

G. S. Gedeon, USA

**3298. Tross, C., Astronomical constants and their importance in lunar trajectory determination,** *ARS J.* **30**, 10, 938–941, Oct. 1960.

Whenever the problem of calculating orbits for space vehicles arises, the accuracy of such calculations receives close scrutiny. No matter what technique is adopted for the orbit calculation, uncertainties enter into the calculations which cannot be eliminated by numerical techniques and mathematical representations. These uncertainties result from insufficient accuracy in the constants utilized for the representation of such effects as Earth's bulge and equatorial radius, the lunar and solar masses and their position coordinates. In this paper an attempt is made to appraise the effects astronomical constants have on lunar vehicle impacts. This study shows that lunar impacts can only be predicted to fall into an area of about 78 miles in radius. More accurate predictions can only be made when the terrestrial and astronomical constants are improved.

From author's summary

**3299. Warzecha, L. W., Performance and design considerations for maneuvering space vehicles,** *Aerospace Engng.* **19**, 11, 18–23, Nov. 1960.

An analysis is made of the required degree of maneuverability for a variety of space missions, including return to the earth's surface. Design and performance penalties associated with high degrees of maneuverability are presented, with particular attention directed toward near-earth orbital flight. Principal conclusions are as follows:

(1) Space maneuvers—Initial space maneuvers will be limited by propulsion requirements and will consist of simple rendevous and intercept in near-earch orbital flight.

(2) Orbital return maneuvers—Vehicle landing or recovery maneuvers will be required for landing at a preselected site from a low-altitude orbit on any chosen day, even though that site may be substantially far away from the satellite orbital plane. In most practical cases, recovery maneuvers will be performed more economically by semi-ballistic aerodynamic glide than by rocket-powered flight.

(3) Vehicles—A modular system for payload and mission flexibility can be achieved without significant weight or performance penalties.

From author's summary

**3300. Detra, R. W., Riddell, F. R., and Rose, P. H., Controlled recovery of nonlifting satellites,** *ARS J.* **30**, 9, 892–898, Sept. 1960.

This paper considers the problems of recovering, at a preselected site, satellites that are orbiting close to Earth (i.e., within a few hundred miles of Earth's surface). Considerations are limited to vehicles which accomplish re-entry into the atmosphere with drag devices. The discussion centers on two possible means of effecting controlled recovery of satellites: Recovery accomplished by a single perturbation of the satellite orbit as may be done with a single solid fuel retrograde rocket; and recovery accomplished by continuous application of a small force of controllable magnitude as may be done by varying the drag area of the vehicle provided it is orbiting in the outer fringes of the atmosphere. These two cases may be considered as extremes in the way in which the perturbing forces can be applied. The "intermediate" case of a controlled recovery by a series of impulsive forces (e.g., by a number of small retrorockets) is not considered.

From authors' summary

**3301. Robinson, R. B., and Morris, Odell A., Aerodynamic characteristics of a model of an inflatable-sphere launching vehicle under simulated conditions of Mach number and altitude,** NASA TN D-640, 12 pp., Dec. 1960.

An investigation has been conducted in the Langley 4- by 4-ft supersonic pressure tunnel to determine the aerodynamic characteristics in pitch of a two-stage-rocket model configuration which simulated the last two stages of the launching vehicle for an inflatable sphere. Tests were made through an angle-of-attack range from $-6°$ to $18°$ at dynamic pressures of 102 and 255 pounds per square foot with corresponding Mach numbers of 1.89 and 1.98 for the model both with and without a bumper arrangement designed to protect the rocket casing from the outer shell of the vehicle.

The results indicated a significant effect of dynamic pressure on the aerodynamic characteristics of the configuration without bumpers which was eliminated by the addition of the bumpers. The bumpers also increased the axial force and provided an increase in stability; however, all configurations investigated were unstable for the moment reference center used.

From authors' summary

## Ballistics, Explosions

(See also Rev. 3267)

**3302. Wood, W. W., and Salsburg, Z. W., Analysis of steady-state supported one-dimensional detonations and shocks,** *Physics of Fluids* **3**, 4, 549–566, July/Aug. 1960.

Authors consider what steady one-dimensional flows can exist behind a steady shock passing through a medium in which one or more chemical reactions can occur and viscosity, diffusion, radiation and thermal conduction may be neglected. They use the Onsager reciprocal relations and the linear flux-force relations of near-equilibrium irreversible thermodynamics. From the behavior of the integral curves of the differential equations it is shown that two classes of solution can occur. The first corresponds to a piston-supported detonation and it is shown to be plausible that this type of solution is unaffected by removal of the piston at a late time. The second class are the "pathological detonations" of Kirkwood and Wood, *J. Chem. Phys.* **22**, p. 1915 (1954), in which the reaction does not reach completion. The use of "frozen" sound speed in the Chapman-Jouguet condition is shown to give solutions which are unstable with respect to small changes in the physico-chemical data. J. Corner, England

**3303. Korobeinikov, V. P., and Riazanov, E. V., A theory of linearized explosion problems including back pressure,** *Appl. Math. Mech. (Prikl. Mat. Mekh.)* **23**, 4, 1066–1080, 1959. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

The problem of a one-dimensional strong point explosion propagating into a medium with variable density is studied. Of particular significance is the inclusion of the undisturbed pressure ahead of the wave. This is contrary to some earlier solutions which neglect this pressure compared to the pressure behind the explosion front. The inclusion of this back pressure makes the problem essentially non-self-similar.

The system of equations is linearized using the ratio of the square of the sonic velocity in the undisturbed gas to the square of the velocity of the explosion front as expansion parameter. The medium is assumed to be a perfect gas. In the proper nondimensional variables it is shown that the resulting system of equations reduces to the system describing the self-similar problem as the expansion parameter goes to zero. Thus a solution is sought with the self-similar solution as the leading term.

A first integral of the obtained system of linearized equations is found and an exact solution is given for a particular law of variation of undisturbed density. With the help of the first integral the system of equations reduces to a system of two linear equations which can be determined by numerical integration. The exact solution is displayed by tables and graphs. The integration for the general solution is not carried out.

This work is representative of the Soviet efforts at obtaining analytical solutions to idealized problems. In so doing the essential parameters are displayed and insights obtained into the nature of the more complicated real phenomena. Reviewer notes that this paper gives more details of the analysis than most recent Russian papers. A. Kovitz, USA

**3304. Lundborg, N., Initiation of liquid explosives in steel tubes exposed to impact,** *Trans. Roy. Inst. Technol., Stockholm* no. 163 (Pure Appl. Math. Phys. no. 9), 22 pp., 1960.

In order to investigate the initiation of liquid explosives by shock, vertical steel-tube containers have been dropped on an anvil. The pressure and flow processes have been studied with water in the tube. From author's summary

**3305. Amster, A. B., Noonan, E. C., and Bryan, G. J., Solid propellant detonability,** *ARS J.* **30**, 10, 960–964, Oct. 1960.

Using a shock sensitivity test, it has been possible to formulate broad patterns of behavior for propellants under certain conditions. Conventional double-base propellants have a sensitivity somewhat less than that of conventional insensitive military explosives. This sensitivity increases slightly with an increase in temperature and markedly with decreasing density. Pore type is important; samples with nonconnected pores increase in sensitivity as temperature decreases. Many composite propellants containing conventional rubber-like binders are not easily detonated. Composites containing high energy binders are readily detonated. An investigation of the attenuation of shock by Lucite has been made. The results of this study make it possible to determine the pressure required to initiate detonation in a propellant under the conditions of the booster sensitivity test.

From authors' summary

## Acoustics

(See also Rev. 3191)

**3306. Williams, J. E., On convected turbulence and its relation to near field pressure,** Univ. Southampton, Dept. Aero. & Astron. Rep. 109, 51 pp., June 1960.

The problem examined is that of analyzing a convected field of turbulence with respect to axes which move with the local flow convection velocity. The general analysis is restricted to homogeneous turbulence convected with a uniform velocity but some of the results are applicable to shear flows. Experimental techniques for measuring the velocity of convection are discussed together with some implications of Taylor's hypothesis. Some properties of the acoustic sources in convected turbulence are considered and particular reference is made to jet mixing regions. Lastly, a crude estimation of the frequency spectrum of noise sources at a certain position in the jet is attempted from observations of the hydrodynamic field close to that point.

From author's summary

**3307. Lyamshev, L. M., Theory of sound radiation by thin elastic shells and plates,** *Soviet Phys.-Acoustics* **5**, 4, 431–438, Apr./June 1960. (Translation of *Akust. Zh., USSR* **5**, 4, 420–427, Oct./Dec. 1959 by Amer. Inst. Phys., Inc., New York, N.Y.)

Author analyzes the problem of radiation of sound by isotropic elastic shells vibrating under the action of harmonic time forces which are statistically distributed over the surface of the shell. An integral relation is derived, connecting the radiation field with the diffraction field of the shell when the latter is free of mechanical forces. Author evaluates the spectral intensity of the pressure field in the Fraunhofer zone for the case of cylindrical and spherical shells, as well as for a thin plate vibrating under the action of statistically distributed harmonic time forces.

Though the author restricts himself to forces which are harmonic in time, the extension to stochastic forces is obvious.

T. K. Caughey, USA

**3308. Zarembo, L. K., A method for determining the width of the front of an acoustic wave similar in form to a sawtooth,** *Soviet Phys.-Acoustics* **6**, 1, 38–41, July/Sept. 1960. (Translation of *Akust. Zh., USSR* **6**, 1, 43–46, Jan./Mar. 1960 by Amer. Inst. Phys., New York, N.Y.)

For linear dissipation processes, and based on spectral analysis of high intensity acoustic saw tooth waves, a method is proposed for determination of the front width of such waves. It can be estimated from absorption data. Discrepancies with experimental data obtained in water are discussed.

F. E. Borgnis, Switzerland

**3309. Khoroshev, G. A., Application of the similarity principle to the study of oscillations excited by cavitation,** *Soviet Phys.-Acoustics* **5**, 4, 485–492, Apr./June 1960. (Translation of *Akust. Zh., USSR* **5**, 4, 472–479, Oct./Dec. 1959 by Amer. Inst. Phys., Inc., New York, N.Y.)

Similarity parameters characterizing noise and oscillations resulting from cavitation near a wall are determined from differential equations of motion for the liquid phase, for the air phase (bubbles in water), for the response of the wall to incident pressure waves, and for the oscillation of a cavitation bubble. Assuming same liquid and wall materials and no effect of changes in Reynolds number or turbulence permits simplification to five similarity criteria. For oscillations in unlimited fluid only three criteria appear: a Euler number (commonly termed the cavitation index or number), a "Stokes" number (ratio of bubble diameter times liquid density to characteristic dimension of flow times air density), and a relative-air-content number. These criteria are utilized to develop relations for scaling the spectral noise composition of cavitation from model results.

Consideration of wall effects are shown to introduce two additional similarity numbers. The first is the ratio of the cavitation noise spectrum to that of the wall natural frequencies of vibration; the second, termed a damping coefficient, is the ratio of the acoustical resistance ($\rho c$) of the fluid to the effective one of the wall at a given frequency. Actually each of these is a family of numbers, one for each frequency. Maintaining the first of these wall criteria constant is shown to eliminate possibility of simulating wall vibrations in a cavitating flow. Immediate need for information on resonance frequencies in upper acoustic region is noted. In low and middle frequency ranges it is concluded that simulation of an increase in vibration level (as at cavitation inception) involves only the three free-fluid parameters.

Reviewer notes possible surface-tension effects on similarity have been ignored, although a Weber number should have evolved from similarity analysis of bubble oscillation equation. Several errors appear due to translator's apparent lack of familiarity with fields involved. J. M. Robertson, USA

**3310. Ribner, H. S., A theory of the sound from jets and other flows in terms of simple sources,** AFOSR TN 60-950 (Univ. Toronto, Inst. Aerophys. Rep. 67), 82 pp. + charts, July 1960.

Author uses the fact that for fluctuating flows of small Mach number in the absence of rigid boundaries the acoustic source is a distribution of simple sources. He amplifies the work of others in this regard and shows its consistency with the well-known equations of Lighthill. His primary purpose appears to be the explanation of the directional character of the radiated sound distribution. It is claimed that this is accounted for by the effects of convection and refraction, and sample calculations are presented to bolster these conclusions.

Reviewer feels that the present method offers a conceptually simpler way of looking at this complex phenomenon. The problems cannot, however, be claimed to be wholly resolved. The problem of convection was treated by Lighthill; the present author has added the effects of time dependence and refraction. Author makes great point of simple sources being nondirective, but by not limiting himself to realizable flows leaves out possibility of spatial correlations enforcing a directivity through the equations of motion. A rather disturbing example in which sound is generated by a frozen pattern of turbulence moving subsonically is presented as explicative of the method. R. Lyon, USA

**3311. Lee, R., et al., Research investigation of the generation and suppression of jet noise,** Gen. Elect. Co., Flight Propulsion Lab. Dept. (Contract NOas 59-6160-c), 45 pp., Jan. 1961.

Analytical and experimental investigations on jet noise suppression have led to establishment of a new theory relating the sound power spectra and distribution of mean flows in the jet field. The method is sufficiently general to be applicable to flow fields of nozzle-suppressors; and is found to be in agreement with test results. A computer program has also been set up for calculating the mean flow characteristics of mixer-type suppressors in general. Scale model testing on various suppressor configurations has indicated that further noise reduction may be achieved using suggested optimization techniques. Directivity investigation on jet noise further confirms that not well-known fact that directivity of suppressors can be predicted with reasonable accuracy based on sound power spectrum data alone. Screech phenomena have been investigated, including flow interference and temperature effects, leading to formulation of a detailed physical model for their generation. From authors' summary

**3312. Canac, F., and Merle, Marie, Vibrations and sound emission of an air jet studied with an ultra-rapid electronic camera** (in French), *C. R. Acad. Sci. Paris* **250**, 10, 1795–1797, Mar. 1960.

**3313. Salceanu, C., and Zaganescu, M., Influence of the nature of the wall of an acoustical resonant tube on the determination of the velocity of sound in liquids** (in French), *C. R. Acad. Sci. Paris* **249**, 23, 2509–2510, Dec. 1959.

**3314. Elberg, S., Measuring the speed of sound in water as a function of the temperature up to 300°C and of the pressure to 150 kg/cm$^2$** (in French), *C. R. Acad. Sci. Paris* **251**, 5, 654–656, Aug. 1960.

**3315. Fisher, I. Z., Temperature dependence of the velocity of sound in dense gases,** *Soviet Phys.-Acoustics* **5**, 4, 470–475, Apr./June 1960. (Translation of *Akust. Zh., USSR* **5**, 4, 459–463, Oct./Dec. 1959 by Amer. Inst. Phys., Inc., New York, N.Y.)

Using modern statistical theory of liquids and dense gases, author analyzes the inversion of the temperature dependence of the velocity of sound in a dense gas. Using a one-dimensional model, author shows that inversion can be predicted without having to include the effects of molecular interactions. In the three-dimensional case, however, author shows that one must include the effects of interaction between molecules in order to predict inversion. T. K. Caughey, USA

**3316. Pippard, A. B., Theory of ultrasonic attenuation in metals and magneto-acoustic oscillations,** *Proc. Roy. Soc. Lond. (A)* **257**, 1289, 165–193, Sept. 1960.

**3317. Salceanu, C., and Huschitt, E., Variation with temperature of the sound velocity in metal bars** (in French), *C. R. Acad. Sci. Paris* **249**, 25, 2731–2733, Dec. 1959.

**3318. Ecollan, J., Hieblot, J., and Rocard, Y., Contribution to a study of silence zones and of long distance focussing of sound** (in French), *C. R. Acad. Sci. Paris* **250**, 22, 3605–3607, May 1960.

## Micromeritics

(See also Revs. 3262, 3266)

**Book—3319. Herdan, G., Small particle statistics; an account of statistical methods for the investigation of finely divided materials,** 2nd rev. ed. New York, Academic Press, Inc., 1960, xxiii + 418 pp. + indices. $14.50.

This is a comprehensive work, intended to serve the needs of research workers in academic and industrial laboratories concerned with finely divided particulate materials. This field is very wide, encompassing ground and milled products, powders, dusts, and

other solid dispersions in gases, atmospheric droplets such as rain and fog, liquid aerosols, suspensions in liquids such as emulsions and biological suspensions, e.g., blood. Characterization of such materials, by particle-size distribution, porosity, specific surface, and other properties, is a problem of statistics, and in the extensive literature on the subject many studies have been devoted to its various aspects. It is a great service to all these disciplines to have the pertinent theories, derivations, and applications assembled in one definitive monograph, organized in a logical and systematic order.

Part I treats particles in the sieve and sub-sieve range, fundamental concepts of statistics, principles of particle-size measurement, distribution of particle size, sampling procedures, forms of distribution functions and their graphical representation, and analysis of differences between determination results. Part II deals with the technological importance of the fineness of solids, correlations and functional relations, dependence of properties upon particle size, shape, porosity and other characteristics of particulate materials, physical and chemical properties as functions of the materials considered as particulate universes. Part III considers the attainment of specified fineness, homogeneity, industrial mixing, and inhomogeneity of molecular weights of polymers. Part IV is devoted to coarse disperse systems, distribution function by weight, bi-variate inequalities, change of distribution with time, and statistical design of investigations.

A large section, Part V, is a detailed exposition of experimental methods and their errors, in particular: microscope method, automatic counting and sizing, sedimentation by gravity and by centrifugal force, radiometric techniques, investigation of dispersions in gases, permeability method for surface area determination, photo-extinction method, and adsorption method.

Each chapter contains extensive and up-to-date list of references, with bibliographic data, but with titles omitted, which detracts somewhat from the value to the user. This slight shortcoming, however, does not negate the great contribution of the author to the literature of particulate materials, a field of ever-growing importance. K. J. DeJuhasz, USA

**3320. Dell, H. A., Hobbs, D. S., and Richards, M. S., An automatic particle counter and sizer,** *Philips Tech. Rev.* **21,** 9, 253–280, 1960.

An instrument for determining the particle-size distribution of particulate materials is described. Authors discuss the general requirements and problems of this type of instrument, giving examples of previous executions. With the present instrument, first a photograph is prepared of the sample to be investigated, and this photograph is scanned by a light spot, termed "flying spot," in a manner similar to that used in television technique. Whenever, in the scanning process, the flying spot encounters a particle image, a signal is given, the duration of which corresponds to the length of chord of the image. A memory circuit prevents counting one particle twice. Sizing is performed by logical circuits which reject pulses shorter than a certain preset length. The necessary initial adjustments, calibration with the aid of artificial samples, and other operational procedures are explained in detail. Some results are given, demonstrating the reproducibility and consistency of the instrument.

Science and technology of particulate materials is in great need of a practical, accurate, and reliable instrument for particle-size determinations; the instrument described, produced by the full resources of a great electronics company, appears to represent an important step toward filling this need.

K. J. DeJuhasz, USA

**3321. Chopin, M., and Chopin, G., On the density measurement of granular and liquid materials** (in French), *C. R. Acad. Sci. Paris* **250,** 3, 480–482, Jan. 1960.

**3322. Koppe, H., Theory of particle size distribution in aerosols** (in German), *Z. Phys.* **156,** 3, 211–216, Sept. 1959.

With the assumption of a pure statistical distribution in space of uniform condensation nuclei, the size distribution of particles in an aerosol has been derived by considering the growth of particles as a nonstationary diffusion process of condensate molecules. A Gaussian distribution is obtained for the deviation of the mass of individual particles from the mean. The standard deviation is proportional with the fourth root of the total volume-fraction occupied by the final particles. J. O. Hinze, Holland

**3323. Simonin, R. F., On stationary molecular waves of spherical form** (in French), *C. R. Acad. Sci. Paris* **250,** 26, 4274–4276, June 1960.

It is proved that the superficial molecules of a liquid jet of uniform flow, which reaches the level of a basin, are transferred to the surface of air bubbles at their origination.

From author's summary

**3324. Simonin, R. F., Molecular surface kinematics** (in French), *C. R. Acad. Sci. Paris* **250,** 10, 1798–1800, Mar. 1960.

Kinematics of surface molecules of a water jet of uniform flow, falling perpendicular to the surface of a basin were studied. Calculations show that the jet depresses the free surface of the basin in a rotational form with constant curvature, that is with minimal surface, and that the depression is covered with surface molecules of the jet with a constant velocity.

From author's summary

## Porous Media

(See also Revs. 2997, 3090)

**3325. Currie, J. A., Gaseous diffusion in porous media: Part 1, A non-steady state method; Part 2, Dry granular materials,** *Brit. J. Appl. Phys.* **11,** 8, 314–324, Aug. 1960.

Part 1 describes an unsteady-state method of measuring gaseous diffusion coefficients. A tube fitted with a katharometer sensing element and containing a mixture of air and hydrogen is suddenly connected to atmosphere via a number of smaller parallel tubes. By assuming the mixture in the larger tube to be stirred, the diffusion coefficient is deduced from the theoretical solution. Values of $D_0$ are in good agreement with other methods.

In Part 2 the parallel tubes are replaced by a cylindrical sample of granular material, giving an apparent diffusivity $D$. For a wide variety of particle types, $D/D_0 = \gamma \varepsilon^{\mu}$ where $\varepsilon$ is the voidage, and $\gamma$ and $\mu$ are constants for each material. This equation does not agree with Burger's formula for electrical conduction through metal containing elliptical inclusions, nor with Bruggeman's equation for heat conduction through the space between spherical particles.

J. F. Davidson, England

**3326. Smirnov, V. A., Mechanism of gas filtration in a three-layered soil** (in Russian), *Inzhener. Sbornik Akad. Nauk SSSR* **28,** 44–50, 1960.

The problem of gas filtration through a sandwich construction consisting of materials of different physical characteristics has been studied. It was assumed that the "sandwich" consists of three-layered soil.

Two cases have been studied: one in which penetration is partial, and one in which the layers consist of completely porous media. Using the potential function the equations describing the motion of gases through each layer are derived and solved, satisfying completely the boundary conditions.

Reviewer is of the opinion that the paper represents a good contribution in the gas dynamic field, describing filtration phenomenology in such a complicated porous system.

M. M. Stanisic, USA

**3327. Saffman, P. G., A theory of dispersion in a porous medium,** *J. Fluid Mech.* **6**, 3, 321–349, Oct. 1959.

Author generalizes the probability approach for longitudinal and transverse dispersion based on the assumption that the velocity distribution over individual canals is proportional to cos $\theta$, $\theta$ being the inclination angle between canal and principal direction of flow. This approach was already treated elsewhere [*Trans. Amer. Geophys. Un.* **39**, p. 67, 1958], but the particular contribution of this study is the elegant introduction of the influence of molecular diffusivity on the residence time $t$ in the canals. Author distinguishes two physical situations $\bar{T} \ll t_1 \ll t_0$ and $t_1 \ll \bar{T} \ll t_0$, for both of which he determines the standard deviations of a particle cloud during transport. Here $\bar{T}$ = average residence in the canals, $t_1$ and $t_0$ are times necessary for a particle to cover distances comparable to radius and length of the canals, respectively, by molecular diffusion only.

A comparison with several tests mentioned in literature verifies the results.

Author indicates how to take account of variations in canal size, and also considers the case when the velocity distribution is proportional to cos $\theta$ to the power $s$, applicable to cases where Darcy's law is not satisfied.

G. de Josselin de Jong, Holland

**3328. Bolotin, V. V., Gavrilov, Iu. V., Makarov, B. P., and Shveiko, Yu. Yu., Nonlinear problems of stability of flat panels for large supersonic speeds** (in Russian), *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk* no. 3, 59–64, May/June 1959.

Effect of aerodynamic loading, as given by the nonlinearized piston theory, at hypersonic speeds on the stability of a flat rectangular panel at zero angle of attack is investigated.

The cases of flow over one side and both sides of the panel are considered. An approximate solution assuming deflections in the form of a twin trigonometric series and solutions using electronic computers are obtained.

Authors arrive at the very important conclusion that panel flutter is possible at Mach numbers smaller than obtained from the linear approximation provided the initial disturbance is large enough. An estimate of the necessary initial disturbance is made and general statements concerning stability of the motion are given.

W. Fiszdon, Poland

**3329. Nielsen, R. F., and Gilchrist, R. E., Transfer of fluid components in a porous medium at constant saturation and fluid velocity,** *J. Petroleum Technol.* **11**, 1, 57–60, Jan. 1959.

When one gas is displaced by another in a porous medium, the volume fraction of one of the gases, $C$, satisfies the partial differential equation $D_tC = -\nu D_xC + mD_x^2C$. If a relatively immobile liquid is present, a term must be added for the interphase transfer. With some restrictions, however, a differential equation for this case (the "wet" case, referring to the presence of oil, in the form of hexadecane, in the experimental work) will be the same, except for a constant factor multiplying $D_tC$. Experimental data for methane displacing $CO_2$ and nitrogen displacing $CO_2$ or methane, in both wet and dry situations, are given. These data are compared with the theoretical solution, and are used to calculate parameters in the solution.

R. E. Gaskell, USA

**3330. Nichiporovich, A. A., and Ryleev, V. I., Results of seepage observations in foundation of some dams** (in Russian), *Gidrotekh. Stroit.* **28**, 1, 20–26, Jan. 1959.

This is a most interesting article for hydrotechnic engineers, since it gives the results of measurements of uplift pressure on five large dams and their powerhouses in the USSR, with hydraulic heads of 13 to 29.8 meters, the structures being founded either on permeable sandy soils or on impermeable sediments. In each individual case the influence of individual parts of the base of the structure on the reduction of uplift pressure is shown. The daring of the structures is considerable, the creep ratio being from 0.135 (in fine sands) to 0.354 (in clays).

It is concluded that concrete cutoff walls decrease the uplift pressure appreciably due to the cutoff wall being pressed on the soil by the horizontal forces, with sufficient force to reduce the soil permeability.

The effect of uplift pressure on the volume of concrete is shown, as well as the favorable influence of drains and aprons.

V. Mencl, Czechoslovakia

**3331. Gusei-Zade, M. A., The calculations for the cover and the base of a stratum when a liquid is in motion in the stratum** (in Russian), *Trudi Moskva Neft In-ta* no. 14, 212–224, 1955; *Ref. Zh. Mekh.* no. 9, 1959, Rev. 10358

An investigation is made of the motion of a homogeneous liquid in a homogeneous stratum with consideration for the permeability of the cover and the base, on the assumption that the liquid in the porous medium is incompressible. The work is divided into two sections. The first deals with the examination of the axially symmetrical problem of the filtration of the liquid from an ideal force well (because of its character and degree of opening) in a finite stratum with different powers and permeabilities of the interlayers situated higher and lower than the relatively weakly permeable bulkhead. The problem is centered on finding the potential of the velocity of filtration $\varphi$ with the following boundary conditions: on the well $\varphi = \varphi_H$ with $r = R_H$, on the feed contour $\varphi = \varphi_0$ with $r = R_0$, the impermeability of the base $\dfrac{\partial \varphi}{\partial z} = 0$ with $z = 0$, the velocity of filtration when the liquid is passing through the bulkhead $\dfrac{\partial \varphi}{\partial z} = \dfrac{k_1}{\mu}\dfrac{p - p_1 - \gamma b_1}{b_1}$ with $z = b$. Here $p_1$ is the pressure in the upper interlayer, $k_1$, $b_1$ are the coefficients of permeability and the power of the bulkhead respectively, $b$ is the power of the lower interlayer, $\mu$, $\gamma$ are the viscosity and specific weight of the liquid respectively. The author shows that the problem is solvable when $\partial\varphi/\partial z \neq 0$, with $z = 0$, that is when the base of the stratum is permeable. A table is furnished for the relative leakages for two cases, when $R_0$ = 1500 and 4500m, while $10^{-2} \geqslant k_1 \geqslant 10^{-6}$ milli-d'Arcy's, $p_1$ = 120 and 100 atm.

The second section is devoted to the analysis of the work of $m$ incomplete wells, draining infinitely long strata. The solution of the problem is found by the method of superposition.

V. A. Karpychev

*Courtesy Referativnyi Zhurnal, USSR*

**3332. Bashkis, K., Filtration through earthen dams fitted with a wedge** (in Russian), *Nauchn. Trudi Lit. S.-Kh. Akad.* **4**, 271–281, 1958; *Ref. Zh. Mekh.* no. 9, 1959, Rev. 10359.

An hydraulic method for the calculation of the filtration of a homogeneous earthen dam fitted with a wedge is described; this wedge cuts through the permeable foundation down to the "vodoupor." By breaking up the body of the dam and its permeable foundation into four fragments (the upper wedge, two fragments in the central section and the bottom fragment) and by using known hydraulic relations (Dupuis, Pavlovskii, Kozen and others) author obtains a system of five equations for the filtration. The problem is investigated for plans dealing with homogeneous as well as nonhomogeneous soils in the body of the plane and the foundation. The data obtained were verified by the author on the

EGDA computer and the confirmation was satisfactory. In addition, the author was able to show that the method of calculation proposed by A. M. Mkhitaryan ["Theory and calculation for filtration through earthen dams," A dissertation, 1951] for the case now being investigated cannot be recommended as it results in a divergence of from 30% to 90%. A. A. Uginchus

*Courtesy Referativnyi Zhurnal, USSR*

**3333. Stepanov, N. G., Calculation for three-dimensional filtration through earthen dams equipped with counterfiltrational devices** (in Russian), *Trudl Kuibychevsk. Inzh.-Stroit. In-ta* no. 5, 309-325, 1958; *Ref. Zh. Mekh.* no. 9, 1959, Rev. 10362.

Paper describes an hydraulic-mechanical system for the calculations of three-dimensional filtration for an earth dam with a water-impermeable diaphragm reaching down to the water-resistant base and with banks built up with homogeneous water-permeable soils having coefficients of filtration identical with those of the soils in the body of the dam. Here the assumption is made that the slopes below water of the dam and the banks are vertical. The author, using the method of conforming reflections, obtains equations for the curve for the depression along the diaphragm and along the water-bed section; this enables an approximate chart to be drawn up for the structural contour. He furnishes graphs to reduce the work on the calculations. In addition, author arranged for some tests to be made in a filtration tank and on EGDA apparatus with three-dimensional models of earthen dams, with the result that a still more simple and purely hydraulic solution for the drawing up of a chart of the structural contour was made available. Comparison of the theoretical construction made by the author with the data of the factual observations for three-dimensional filtration furnishes proof of the adequate coincidence of the general picture of the filtration obtained by the theoretical and the experimental methods. A. A. Uginchus

*Courtesy Referativnyi Zhurnal, USSR*

**3334. Pirverdyan, A. M., and Babich, E. S., An approximate method for the calculation of the inflow of liquid to a round battery of wells in an elastic regime of filtration** (in Russian), *Trudi Azerb. Nauk-i In-ta po Dobyche Nefti* no. 7, 111-117, 1958; *Ref. Zh. Mekh.* no. 9, 1959, Rev. 10364.

The filtration of an elastic liquid can be described in an approximate fashion by means of two depression eddies of pressure, diverging inwardly and outwardly from the round battery of wells. By making use of some elementary expressions for the conservation of mass of a flowing liquid the authors find an approximate solution for the given problem. V. P. Pilatovskii

*Courtesy Referativnyi Zhurnal, USSR*

**3335. Chernov, B. S., Investigation of gusher and fluid input wells without stoppage and the obtaining of curves for the re-establishment of pressure for the determination of the parameters of the stratum** (in Russian), *Trudl Vses. Neftegaz. Nauk.-i. In-ta* no. 17, 162-184, 1958; *Ref. Zh. Mekh.* no. 9, 1959, Rev. 10366.

Comparison of the results of the investigation of wells obtained by the changes of the regimes of working with the results of investigations obtained by complete stoppage of the wells gives good convergence. Because of this the possibility emerges of being able to obtain curves for the re-establishment of pressure without stoppage of the wells. On the assumption that, for the comparatively short time taken for the examination of the wells (6 to 7 hours), the influence of the boundaries of the stratum and of the surrounding wells would not affect the picture of the re-establishment curve, provided that during the process of examination and for some little time previous to the examination there was no change in the regime of work of the surrounding wells, the author considers the possibility of utilizing the formula for the distribution of pressures, obtained for the case of a single well (an axially symmetrical problem) situated in an infinite homogeneous horizontal elastic stratum, with the object of determining the parameters of the stratum. Two cases are investigated for approximate evaluations of the pressure in the stratum. The first case, the evaluation of the stratum's pressure at the oil's originating sources, the result of prospecting for a small number of wells; the second for sources involving the exploitation of a large number of wells. The method of investigation of wells without their being closed down is illustrated for the whole operation by means of a number of examples. M. V. Filinov

*Courtesy Referativnyi Zhurnal, USSR*

**3336. Trebin, G. F., Influence of the interaction of liquids and gases with the surfaces of porous media on their permeability** (in Russian), *Trudl Vses. Neftegaz. Nauk-i. In-ta* no. 15, 122-136, 1958; *Ref. Zh. Mekh.* no. 9, 1959, Rev. 10378.

Paper consists of an investigation of possible changes in permeability because of the influence of adsorptional layers developing on the surface of rock-forming particles and because of the swelling of the clayey material contained in the rock. An experimental evaluation of the influence of these factors on the permeability, undertaken by the author, who made a whole series of assumptions, indicated that it changes very inappreciably. Experimental investigations carried out by the VNII in the laboratory for the study of the physics of strata are described. A detailed account is given of the methods employed for saturating the porous media with water, for the cleansing of the water from the fine mechanical admixture, and for the filtration with distilled water. Plans of the apparatus used for the saturation and the filtration are given. Descriptions are also given in similar detail of the apparatus used for the saturation and filtration of hydrocarbon liquids through porous media. The results of the investigations are put forward in the form of tables and graphs. These show that there is no appreciable change in the permeability due to the influence of the adsorptional layers. The permeability remains also practically invariable over a period of time and when distilled water is being filtered through natural porous samples with known quantities of known types of clayey fractions.

M. V. Filinov

*Courtesy Referativnyi Zhurnal, USSR*

**3337. Horvay, G., Flow of liquid metal past a porous flat plate,** *ASME Trans.* **82** E (*J. Appl. Mech.*), 4, 749-750 (Brief Notes), Dec. 1960.

A simple boundary-layer approximation formula is derived for the temperature distribution in liquid metal which flows past a porous flat plate at zero incidence at velocity $U$ and is sucked into it at velocity $V$. From author's summary

## Geophysics, Hydrology, Oceanography, Meteorology

(See also Revs. 2910, 2988, 3084, 3178, 3244, 3280, 3322, 3363)

**3338. Healy, J. H., and Press, F., Two-dimensional seismic models with continuously variable velocity depth and density functions,** *Geophysics* **25**, 5, 987-997, Oct. 1960.

A laminated model consisting of alternate layers of aluminum and epoxy resins is used in order to simulate inhomogeneous media, where velocity and density depend on depth.

A first set of experiments deals with disks 24 inches in diameter and 1/16 inch thick. Authors say source and receiver consist of titanate cylinders 1/4 inch × 1/8 inch, and at the same time consider plane waves propagating in a direction perpendicular to disk

axis. Reviewer cannot see how such plane waves can be generated with such small sources. Anyhow, good experimental check is claimed for compressional and shear waves.

Another experiment simulates Rayleigh waves around a laminated disk representing earth's crust. Measured phase and group velocities for periods extending from 10 to 60 microseconds agree with theory.
J. M. Loeb, France

**3339. Ecollan, J., and Rocard, Y., Microbarographic signals in connection with the great earthquake in Chili** (in French), *C. R. Acad. Sci. Paris* **251**, 4, 523-525, July 1960.

**3340. Rocard, Y., On the long period seismic signal obtained in the nuclear test of Reggane** (in French), *C. R. Acad. Sci. Paris* **250**, 12, 2244-2246, Mar. 1960.

**Book—3341. Yevdjevich, V. M., Flood routing methods, discussion and bibliography,** Washington, U.S. Geological Survey, Surface Water Branch, Water Resources Division, 1960, 136 pp.

Work contains a brief review of methods used for determination of unsteady flow in channels and reservoirs, and an extensive bibliography of 736 items. Summary of theory of unsteady flow, its definitions and limitations are very lucidly explained in the introduction. Different flood routing methods are compiled and problems for study disclosed. The bibliography is of greatest value, and particularly annotations with brief contents of every item. All papers are given in chronological order, beginning with I. Newton's "Principia," of 1687. Titles of magazines are given abbreviated (index omitted!), some are translated.

This bibliography was prepared for Soil Conservation Service, but it will be used by many hydrologists and hydraulic engineers with great appreciation.
S. Kolupaila, USA

**Book—3342. Serebriakov, V. V., Book of problems in river hydraulics [Zadachnik po rechnoi gidravlike],** Moscow, Rechnoi Transport, 1959, 151 pp. $0.55.

A brief review of hydraulics with series of solved typical problems. More problems are given unsolved, and answers are added at the end, with some hints for their more complex solutions. Booklet is prepared for use in secondary technical schools. Part of the problems are compiled from other textbooks. All problems are directed to practical hydraulic engineering.
S. Kolupaila, USA

**Book—3343. Chebotarev, A. I., General hydrology [Obshchaia gidrologiia],** Leningrad, Gidrometeoizdat, 1960, 540 pp. $2.80.

A modern textbook of continental hydrology (excluding oceanography) for university students contains general problems, natural water cycle and heat balance, precipitation and evaporation, ground waters, rivers, lakes, swamps, glaciers, and their laws. Major part is dedicated to river runoff, water flow and sediment transport, bed formation and development. Observations and measurements are emphasized in every chapter. Book ends with a history of hydrologic investigations in Russia, and a bibliography, exclusively of Russian books and papers. Book gives a good cross section of present state of hydrological investigations in Russia.
S. Kolupaila, USA

**3344. Vladimiersku, I., On the analysis of flow discharges from small watersheds by "index hydrograph"** (in Russian), *Rev. Mech. Appl.* **4**, 3, 449-462, 1959.

The review is made from a Russian translation of a Roumanian paper: "Studii Si cercetări de mecanică aplicată, Acad. R.P.R.," **9**, 4, 1958.

The "index hydrograph" is defined according to Cuenod, *Houille Blanche* no. 3, 1956, as a time function of precipitation of constant intensity of infinite duration. A time factor entering in the equation is a characteristic of the watershed. Natural river flow hydrographs, however, do not coincide well with the Cuenod equation. The author modifies this equation and obtains a favorable comparison with natural hydrographs. Author then obtains solutions for constant intensity of precipitation with finite duration, for linear variation of precipitation from zero to the maximum and from maximum to zero. Any graph of precipitation might be decomposed to these three simple cases and each treated separately. By the principle of superposition the hydrograph for actual precipitation is then obtained. The graphs represented in the paper for a river in Switzerland show a very satisfactory coincidence of gaging station data with computed hydrographs. The method does not require a long computation procedure. For its wide application, field measurements are necessary in order to determine the time factor for typical watersheds. The author mentions possibilities of research using models of watersheds, which may bring new interesting developments in the problem treated.

The reviewer's opinion is that this paper has merit primarily due to its theoretical approach to the flood runoff problems. However, since Dolgov apparently first published in 1914 his theory of runoff, the application of the theory requires local watershed characteristics, as in this paper, which in most cases are not available. The basin time factor must be known in the author's method. However, it might be determined apparently from available field data on similar basins and then the method can be applied. Its practical significance is in the fact that formulas are developed for three basic precipitation distributions, which make it possible to treat any precipitation graph by its separation into these three simple shapes. The reviewer believes that watershed models, mentioned by the author, are a promising method of investigation of the runoff. Coefficients depending upon the topography of the watershed might be obtained, as well as characteristics for typical basins.
B. S. Browzin, USA

**3345. Long, R. R., A laminar planetary jet,** *J. Fluid Mech.* **7**, 4, 632-638, Apr. 1960.

Some surface and subsurface currents in oceans tend to concentrate in narrow strips. Velocity concentrations also occur in the atmosphere and in certain laboratory experiments. Author has previously [*J. Meteor.* **9**, 187-99, 1952] studied experimentally the motion of shell of water between two rigidly connected concentric hemispheres in rotation. An obstacle is moved along latitude circle in direction of rotation, but either slower or faster. If obstacle moves slower than fluid, a series of waves develops around the globe. If obstacle moves faster than fluid, it drags around a jet. Author assumes two-dimensional motion parallel to spherical surfaces and with no variations in directions normal to these surfaces. Navier-Stokes and continuity equations are written in polar coordinates, with introduction of stream function. Some approximations are introduced from boundary-layer theory. Linear case is solved. Author draws some inferences about nonlinear case by dimensional considerations.
A. Balloffet, USA

**3346. Larras, J., and Claria, A., Research on the relative action of waves and wind** (in French), *C. R. Acad. Sci. Paris* **250**, 10, 1801-1802, Mar. 1960.

**3347. Karol, I. L., A semi-empirical theory of vertical turbulent diffusion in the boundary layer of the atmosphere** (in Russian), *Inzh.-Fiz. Zh.* **3**, 4, 54-64, Apr. 1960.

Kolmogorov, Obukhov, Landau, Loyzinsky, Fedyaevsky and others have created a Soviet school of boundary-layer and turbulence analysts of world-wide fame. The article in review is in the tradition of that school. Karol uses a Laplace transform technique for solving the parabolic differential equation of nonsteady vertical turbulent diffusion of aerosol concentration within the

atmospheric boundary layer. The initial conditions are those of an instantaneous point-source at a height $H$ and boundary conditions are satisfied at infinity and at the edge of the "roughness layer." The solution is a limit-form solution when the height of the edge of the roughness layer tends to ground level. It is given as a pair of asymptotic expressions for the vertical distribution of the aerosol concentration and for the aerosol flux at ground level, for a wide range of values of $H$ and of time.

G. A. Tokaty and M. J. Cohen, England

**3348. Opik, E. J., and Singer, S. F., Distribution of density in a planetary exosphere,** *Physics of Fluids* **2**, 6, 653–655, Nov./Dec. 1959.

A new theory giving the distribution of density with altitude for an exosphere was developed. It was based not on the assumption of the local thermodynamic equilibrium, but on the consideration of the statistical distribution of particle orbits issuing from the base of the exosphere and the integration of their contribution to the density of each level. Numerical calculation by the new method was carried out for the case where the field of force is only gravitational. Results gave values considerably lower than those conventionally calculated by the hydrostatic equation. However, nothing was mentioned about comparison with the observation.

K. Miyakoda, Japan

**3349. Golubovskis, E., Determination of the deflection radius of a river-bed in relation to the soil's stability** (in Russian), *Izv. Akad. Nauk LatvSSR* no. 6, 129–134, 1958; *Ref. Zh. Mekh.* no. 9, 1959, Rev. 10127.

The stability is investigated of the transverse profile of a river-bed on a curvilinear portion of it. As a criterion for the displacement of the soil is taken the force exercised by the flow (see M. V. Potapov) which is equal, for the longitudinal and transverse directions respectively, to

$$S = \gamma H i, \qquad S = \gamma H I \qquad \left(I = \frac{v^2}{gr}\right)$$

where $i$ and $I$ are the longitudinal and transverse slopes of the surface. The determining spot for the erosion of the soil is taken to be the base of the external bank, where the stability of the soil decreases proportionally to the cosine of the angle of declination of the bank to the horizon. By assuming that the longitudinal and transverse "drawing" forces are equal in magnitude and by carrying out their vectorial addition, the author obtains an expression for the total "drawing" force. For the purpose of taking into account the other factors influencing the "drawing" force of the flow, a coefficient is introduced into the calculation formula, for which approximate experimental values are given in terms of the relation of the depth of the flow to its width. Attention is drawn to the fact that further work has to be done to give added precision to this coefficient. From the formula obtained it was found possible to determine the minimum radius of the river-bed's deflection in relation to the velocity of the flow, its depth and to the transverse profile of the river-bed, while taking into consideration the stability of the soil or by finding the "drawing" force of the flow on the outer bank and on the bottom, and in combination with the above to determine the necessary type of river protection works. (See M. V. Potapov, Collection of papers, Vol. 1, 1950, p. 306.)

O. L. Yushmanov

*Courtesy Referativnyi Zhurnal, USSR*

**3350. Gruza, G. V., On the zonal characteristics of macro-turbulence** (in Russian), *Izv. Akad. Nauk UzSSR, Ser. Fiz.-Matem. Nauk* no. 2, 57–67, 1958; *Ref. Zh. Mekh.* no. 9, 1959, Rev. 10318.

Calculations are given for a series of zonal characteristics of macro-turbulence on the data furnished in charts AT–700 and OT–500/1000 for January to July, 1953. In making these calculations formulas proposed by A. S. Monin [*Izv. Akad. Nauk SSSR, Ser. Geofiz.*, no. 4, 452–463, 1956] were used which took in the calculations for other months of the year (September-October). The mean monthly zonal values for AT–700, OT–700/1000 are given, as well as the index of circulation, the zonal velocity, the component of the correlational tensor for the velocity fields $\overline{v_\lambda'^2}$, $\overline{v_\theta'^2}$ and $\overline{v_\theta'}, \overline{v_\lambda'}$, the kinetic energy of the macro-turbulence, the magnitude of the transformation of the energy of the zonal motion into the energy of macro-turbulence, the macro-turbulent flow of heat along the meridian, the coefficient of exchange for the heat $K_T$ and, finally, the divergence of the macro-turbulent flow of heat. A marked correlation was noted between the values of $K_T$ and $\overline{v_\theta'^2}$. Isopleths are also given (that is isolines in the plane latitude-time) as well as a series of zonal characteristics, enabling observations to be made of the phasal shears between the changes with time. It was established that the index of circulation on a given latitude and the divergence of the momentum of flow along the meridian (that is the magnitudes $\overline{v_\theta'}\ \overline{v_\lambda'}$) vary almost synchronously.

A. S. Monin

*Courtesy Referativnyi Zhurnal, USSR*

**3351. Borisenkov, E. P., Horizontal divergence of velocity in the atmosphere** (in Russian), *Meteorol. i Gidrol.* no. 6, 3–7, 1958; *Ref. Zh. Mekh.* no. 9, 1959, Rev. 10325.

The term "horizontal divergence" is not quite precise inasmuch as divergence is a scalar magnitude. The question discussed is the divergence of horizontal velocity. It is shown that direct determination of divergence for large-scale atmospheric motions in accordance with the actual wind data recorded leads to exaggerated values by the modulus for the divergence. This fact can be explained by the large effect of errors when making observations of the wind. As an example results are given for the divergence calculations using wind data for the 500-mb level. It is demonstrated that not only the magnitude but the sign of the calculated value depends essentially on the magnitude of the step taken when substituting end differences for the derivatives and in the direction of the coordinate's axes. The transition to the mean values for divergence over the whole depth of the atmosphere, calculated by means of the wind data on the principal isobaric surfaces, does not result in any appreciable diminution in the influence wielded by observational error. As an example a chart is given for the divergence values averaged by the depth of the atmosphere, from which it will be seen that the order of magnitude of the mean divergence worked out by the above method is just the same as for the divergence on a fixed isobaric surface. An analysis is made of the influence of error in measuring the magnitude and direction of the wind in accordance with the results of determining the divergence by the formula recorded in a natural system of coordinates. It is shown that with existing standards of accuracy for the determination of winds at altitudes when the probable error in regard to the wind direction may be ±6 to 10° and the velocity error one of ±10 to 20%, the velocity of divergence is determined with the probability of error of the order of 100 to 200%. This conclusion is supported in a numerical example. It is for this reason that "The manual for short-term prognoses for the weather," Part 1 [Gidrometeoizdat, 1956] overestimates the value of the role filled by the so-called divergence term in the vortex equation.

L. S. Gandin

*Courtesy Referativnyi Zhurnal, USSR*

**3352. Utina, Z. M., Consideration of the differences in the coefficient of turbulence for temperature and for humidity in the problem of joint transformation** (in Russian), *Trudi Gl. Geofiz. Observ.* no. 69, 41–44, 1957; *Ref. Zh. Mekh.* no. 9, 1959, Rev. 10327.

The equation for the turbulent diffusion of a substance $s$ is recorded in the form

$$\bar{u}\frac{\partial \bar{s}}{\partial x} = \frac{\partial}{\partial z} k \frac{\partial \bar{s}}{\partial z}$$

where the overhead lines indicate an average value by time; this is large in comparison with the periods of turbulent pulsations and small in comparison with the characteristic time of variations in magnitudes $\bar{u}$ and $\bar{s}$. It is assumed that in taking the mean in this manner the coefficient of turbulent diffusion $k$ is the same for different substances. For large intervals of time during which $\bar{u}$ and $\bar{s}$ change it is proposed to average the equation for diffusion a second time, which results in obtaining the following

$$\bar{u}\frac{\partial \bar{s}}{\partial x} = \frac{\partial}{\partial z} \bar{k} \frac{\partial \bar{s}}{\partial z}$$

where $\bar{k}$ is the mean value of $k$ in suspension by the magnitude $\partial \bar{s}/\partial z$. The $\bar{k}$'s of this type will already differ for various substances. The data of the Pakhta-Aral'sk expedition GGO give the value of $\tilde{k}_T$ as equal to 0.03 m²/sec for the temperature and $\tilde{k}_e$ = 0.07 m²/sec for the humidity. This difference is explained by the fact that extensive gradients of temperature were observed in the night inversions when the turbulent exchange was weak, while the large gradients of humidity were day phenomena, accompanied by violet turbulence. In considering the difference between $\tilde{k}_T$ and $\tilde{k}_e$ an equation was found for the problem of the joint transformation of temperature and humidity under the influence of irrigation and the setting for this problem following the method given in D. L. Laichtman's article [title source no. 39, 239-240, 1953].

A. S. Monin

*Courtesy Referativnyi Zhurnal, USSR*

**3353. Shishkova, I. A., A procedure for computing local accelerations** (in Russian), *Trudl Tsentr. In-ta Prognozov* no. 61, 111-119, 1957; *Ref. Zb. Mekb.* no. 9, 1959, Rev. 10333.

Calculations are described for determining the local accelerations $\partial u/\partial t$ and $\partial v/\partial t$ in conformity with the initial field of velocities. As the starting point the equations of motion are used including the viscous terms. The equations of motion with adoption of a quasigeostrophic approximation in a system of coordinates $x$, $y$, $p$, $t$, computed relative to the local accelerations, take the form of

$$\frac{\partial u}{\partial t} = l\left(v - \frac{1}{l}\frac{\partial H}{\partial x}\right) - \frac{1}{l^2}\left(\frac{\partial H}{\partial y}\frac{\partial^2 H}{\partial x \cdot \partial y} - \frac{\partial H}{\partial x}\frac{\partial^2 H}{\partial y^2}\right) - \tau\frac{\partial u}{\partial p} + k\frac{\partial^2 u}{\partial p^2}$$

$$\frac{\partial v}{\partial t} = -l\left(u + \frac{1}{l}\frac{\partial H}{\partial y}\right) + \frac{1}{l^2}\left(\frac{\partial H}{\partial y}\frac{\partial^2 H}{\partial x^2} - \frac{\partial H}{\partial x}\frac{\partial^2 H}{\partial x \cdot \partial y}\right) - \tau\frac{\partial v}{\partial p} + k\frac{\partial^2 v}{\partial p^2}$$

Here $H$ is the absolute geopotential, $l$ Coriolis parameter, $\tau = \partial p/\partial t$ is the vertical velocity, $u$ and $v$ the horizontal components of velocity on axes $x$ and $y$, $k$ is the coefficient of turbulence. In these equations, after conversions with the aid of some substitutions, the special derivatives for the calculations were replaced by end differences. As the result of his computations the author's deductions were: (1) It is possible to judge the direction of the change of wind toward the high or low pressure areas in the course of time by means of the sign for $\partial v/\partial t$ with a certainty of 76 to 78%. (2) Should the signs for $\partial v/\partial t$ be identical at a given point by altitude then the changes of wind will take the same direction at all levels. Should the signs for $\partial v/\partial t$ be different, then the changes of wind direction at different levels may be contrary.

E. P. Borisenko

*Courtesy Referativnyi Zhurnal, USSR*

**3354. Giao, A., and Roulleau, J., On the variation with height of the mean vertical gradient in the free atmosphere** (in French), *C. R. Acad. Sci. Paris* **250**, 5, 896-898, Feb. 1960.

**3355. Roy, M., Experiment with a missile for high atmosphere research** (in French), *C. R. Acad. Sci. Paris* **250**, 21, 3431-3432, May 1960.

# Naval Architecture and Marine Engineering

(See also Revs. 3064, 3065, 3084, 3136)

**3356. Eggers, K., Increase in resistance in swells through energy considerations** (in German), *Ing.-Arch.* **29**, 1, 39-54, 1960.

Shiplike bodies, moving forward and undergoing heaving motions in a regular train of waves, are considered. Bodies are represented by source-sink distributions, which satisfy linearized boundary conditions on the surface, and doublet distributions of harmonically varying intensity which take into account the variation of the displacement due to the waves encountered.

Energy and momentum relations for the resulting potential-flow field are systematically applied to obtain expressions for the coefficients of resistance to forward motion and of damping of heaving motions for various frequencies and speeds of advance for both head and following seas. Results of numerical calculations are presented and shown to be in reasonable agreement with experiment.

L. Landweber, USA

**3357. Balquet, R.-J., On the problem of the substitution of the submerged part of a vessel by a scale model for the calculation of the frictional resistance** (in French), *C. R. Acad. Sci. Paris* **250**, 23, 3773-3775, June 1960.

**3358. Judd, P. H., Longitudinal strength and vibration of ships by electronic computer,** *N. E. Coast Instn. Engrs. Ship. Trans.* **77**, 2, 53-76, Dec. 1960.

The object of this paper is to demonstrate the advantages of using a digital computer for routine ship design calculations. Two particular programs are taken as examples. First a general outline is given of the approach to programming a calculation, and a review made of the factors to be borne in mind regarding its final properties and scope.

From author's summary

**3359. Dorey, Mary W., UNIVAC processing of the electronic failure tabulation problem for the Bureau of Ships,** David W. Taylor Mod. Bas. Rep. 1433, 104 pp., Nov. 1960.

**3360. Gesswein, Barbara H., and Moses, F., Calculated modes and frequencies of hull vibration of USS George Washington (SSBN 598),** David W. Taylor Mod. Bas. Rep. 1464, 23 pp., Nov. 1960.

Normal mode shapes, natural frequencies, and bending moments of vertical flexural vibration and of longitudinal bar-type vibration of the hull and of the shafting system were calculated for USS GEORGE WASHINGTON (SSBN 598). The methods used in evaluating the parameters necessary for the calculations are described.

From authors' summary

**3361. Sommet, J., On the calculation of the virtual mass of a vessel, excited harmonically in a finite depth** (in French), *C. R. Acad. Sci. Paris* **250**, 26, 4280-4282, June 1960.

**3362. Balquet, R.-J., On the inner displacement of quantity of a liquid disturbed by the passage of the submerged part of a vessel** (in French), *C. R. Acad. Sci. Paris* **250**, 22, 3570-3572, May 1960.

**3363. Venning, E., Jr., Applicability of a supercavitating propeller to a small speedboat,** David W. Taylor Mod. Bas. Rep. 1459, 25 pp., Nov. 1960.

A supercavitating propeller was designed and built to operate under conditions that might exist on a small speedboat. This propeller was later compared against two other types of speedboat propellers in laboratory tests. Computations were then made to predict the performance of a standard propeller. Comparison of the test results with the predicted performance of the standard propeller indicated that the supercavitating propeller was only equal to the standard propeller in efficiency under the conditions specified. From author's summary

**3364. Lang, T. G., and Daybell, Dorothy A., Watertunnel tests of a base-vented hydrofoil having a cambered parabolic cross section,** NAVWEPS Rep. 7584 (U. S. Nav. Ord. Test Sta., China Lake, Calif., TP 2569), 59 pp., Oct. 1960.

Results of water-tunnel tests on a base-vented hydrofoil having a cambered parabolic cross section show lift-to-drag ratios of 24 for an infinite aspect ratio and 8.5 for an aspect ratio of 1.44 at the minimum ventilation numbers attained.

The drag coefficients measured are in good agreement with linearized cavity theory. The measured lift and moment coefficients are in agreement with wind-tunnel tests of airfoils with streamlined cross sections having the same camber line.

The air cavity produced behind the base does not spring forward to the leading edge unless the hydrofoil has stalled or excessive cavitation occurs behind the leading edge.

From authors' summary

**3365. Fabula, A. G., Theoretical lift and drag on vented hydrofoils for zero cavity number and steady two-dimensional flow,** U. S. Nav. Ord. Test Sta., China Lake, Calif., Rep. 7005, TP 2360, 46 pp., Nov. 1959.

Linearized, plane cavity-flow theory for steady, two-dimensional flow is used to predict lift and drag on hydrofoils with forced ventilation by gas exhaust on one side. Conditions assumed include cavity pressure equal to free-stream pressure, smooth separation of the free streamlines, and infinite cavity length. Lift and drag coefficients are calculated versus exhaust point for several basic profile shapes, and formulas for lift, drag and moment coefficients are given for arbitrary profile. The infinite cavity length assumption is contradicted by the implied cavity wall contours for certain parameter ranges, and a finite cavity analysis is then more appropriate. The infinite and finite cavity solutions are compared for a simple vented strut. A limited comparison of calculated lift coefficients with experimental data is favorable. More detailed comparison will be made in a later report for arbitrary cavity pressure. From author's summary

**3366. Sommet, J., Calculation and experimental determination of the virtual mass of the submerged part of a vessel excited harmonically in a finite depth** (in French), *C. R. Acad. Sci. Paris* **251**, 1, 35–37, July 1960.

**3367. Cumberbatch, E., Cavitating flow past a large aspect-ratio hydrofoil,** Calif. Inst. Technol., Engng. Div. Rep. 47-12 (Contract Nonr 220(12)), 17 pp. + figs., May 1960.

Tip effects on the cavitating flow past a large aspect-ratio lifting hydrofoil are considered. The tip vortices arising from the flow leakage around the tip from the lower to the upper side of the hydrofoil are assumed to cavitate. The flow over the central section of the hydrofoil is taken as two-dimensional cavity flow and hence there is a wide planar cavity there. The separate cavity regions are taken not to coalesce. The flow is represented by a simple horseshoe-vortex model and descriptions of the flow over the central section, near the tip and well downstream, are derived and appropriately matched. The lift on the hydrofoil is then calculated, taking the downwash into account. The lift is seen to be reduced by the tip effects, and shows good comparison with experimental results. From author's summary

**3368. Biarez, J., and Negre, R., Study of the rotation of a stiff vertical bulkhead subject to a reversing momentum** (in French), *C. R. Acad. Sci. Paris* **250**, 14, 2513–2515, Apr. 1960.

**3369. Wilson, B. W., Characteristics of anchor cables in uniform ocean currents,** Texas A & M, Dept. Oceanogr. and Meteor. TR 204-1, 157 pp., Apr. 1969.

This report treats an exact solution of the equilibrium in a fluid of a flexible mooring cable of uniform line density subject to a steady current whose velocity is constant with depth. Re-examination of experimental data for towed cables justifies the description of the hydrodynamic drag forces in terms of normal and tangential components of the velocity with respect to the cable and indicates the effects of roughness or stranding of the cable. The analysis includes the full effect of these forces. The general solution is shown to yield previously known solutions for particular cases, such as that of the catenary; it checks the special cases studied by Landweber, Dove, Zajac and Pode. Results are presented in dimensionless form in figures and tables and define the scope (cable length/depth), stance (horizontal projection/depth), relative tensions (surface or bottom tension/depth × cable weight in water per unit length) in terms of the angles of inclination with the horizontal of the cable at the upper and lower tiepoints. An example is given of the application of results to the mooring of ships in a tideway. From author's summary

## Friction, Lubrication and Wear

(See also Revs. 3022, 3072, 3259)

**3370. Drutowski, R. C., and Mikus, E. B., The effect of ball bearing steel structure on rolling friction and contact plastic deformation,** *ASME Trans.* **82** D (*J. Basic Engng.*), 2, 302–308, June 1960.

Retained austenite content had different significance depending on whether low or high contact stresses were imposed. Using a ball rolling between flat plates, rolling friction and contact plastic deformation were measured for a steel ball against steel plates of the same structure as the ball, and a cemented carbide ball against steel plates having the various structures. Material was 52100 steel, heat treated to have 0, 3.9, 7.4, and 18.4% retained austenite, respectively. Hardness ($R_c$) was 58, 64, 62, and 59, respectively. Micro-elastic limit was only 58,000 psi for the third treatment, against 123,000 and 127,000 for the first two. Rolling friction decreased with decreasing retained austenite and was least for the structure containing none. For instrument bearings having low contact stresses, a structure containing no retained austenite is recommended for maximum elastic limit, dimensional stability, maximum resistance to plastic deformation, and minimum rolling friction. At high stresses, on the other hand, the hardest structure shows the least macroplastic deformation.

E. A. Ryder, USA

**3371. Reichenbach, G. S., The importance of spinning friction in thrust-carrying ball bearings,** *ASME Trans.* **82** D (*J. Basic Engng.*), 2, 295–301, June 1960.

Friction was measured while rolling balls on flat plates and in V-grooves. Plate loaded by 3 balls above and 3 below was reciprocated for a distance of one inch at 6 to 600 strokes per hour. Variables studied were contact angle, conformity, load, lubricant and temperature. Rolling moment of ball on a flat plate was between 1 and 2 orders of magnitude less than that of ball in grooves of various shapes. While different types of oil or no oil at all

gave same force reading on flat plates, author states "the use of V-grooves proved to be quite a sensitive method for determining the friction properties of a variety of oils." Repeatability was good. Spin-friction coefficient was evaluated analytically for a series of tests with various grooves (7808 oil) and was generally in the neighborhood of 0.07. Important conclusion (by author) is that in many types of rolling contact the major part of rolling resistance is due to spinning of the balls. An example is given of a jet-engine thrust bearing in which essentially all of the heat generated was accounted for by spin friction.

E. A. Ryder, USA

**3372. Shaw, M. C., Ber, A., and Manin, P. A., Friction characteristics of sliding surfaces undergoing subsurface plastic flow,** *ASME Trans.* **82** D (*J. Basic Engng.*), 2, 342-346, June 1960.

The influence of subsurface flow upon the coefficient of sliding friction is discussed. A simple test procedure for studying the friction characteristics of sliding metal surfaces, one of which is being subjected to plastic flow in bulk, is described, and representative data are presented for both dry and lubricated sliding.

From authors' summary

**3373. Augustin, J. U., Friction and lubrication in engines, in particular with added colloidal graphite** (in German), *Wear* **3**, 2, 114-121, Mar./Apr. 1960.

The corrosive action of HP additives and moisture on the surface of cast-iron cylinder walls and the vulnerability of the Beilby layer are discussed. Protection of surfaces by colloidal graphite has been studied in laboratory and field tests. HP additives do not interfere with the beneficial effects of graphite. Graphitized cylinder walls of an Otto-motor were exposed to unfavorable climatic conditions in the laboratory; corrosion resistance was higher than that of a motor protected only by an anti-corrosion oil.

From author's summary

**3374. Rubenstein, C., The influence of workhardening on the coefficient of friction,** *Wear* **3**, 2, 150-153 (Brief Note), Mar./Apr. 1960.

Friction tests with soft metals (e.g. when a steel ball slides on indium) yield coefficients of friction which are dependent on load, reaching high values at low loads. Author [*Wear* **2**, p. 85, 1958/59] referred this behavir to the workhardening of material in the contacting regions. The friction coefficients of gold, platinum, steel and tin studied by J. S. Courtney-Pratt and E. Eisner [*Proc. Roy. Soc. Lond.* (*A*) **238**, p. 529, 1957] are independent of load. Author attributes this to the fact that these metals are fully workhardened when sliding in the contact regions begins. This explanation differs from the opinion of D. Tabor [*Proc. Roy. Soc. Lond.*, (*A*) **251**, p. 378, 1959].

H. F. K. Drescher, Germany

**3375. Porgess, P. V. K., and Wilman, H., The dependence of friction on surface roughness,** *Proc. Roy. Soc. Lond.* (*A*) **252**, 1268, 35-44, Aug. 1959.

The variation of the coefficient of friction (0.57 to 0.88) with surface roughness is investigated experimentally in the case of emery paper sliding on various grades of emery paper. It is established that the main variable part of the friction is a function of the ratio of the radii of the emery particles in the two surfaces, and is a minimum when the radii are equal. Theoretically it is shown that the work done in the repeated up-and-down motion of the smaller particles against the elastic backing, as they slide over the larger particles in the opposing sheet, accounts well for the form and magnitude of this friction component, the fraction of the energy lost in the hysteresis being 0.6. The remaining practically constant part of the friction is then 0.42, which is appreciably larger than the friction of sapphire on sapphire, 0.2, but is not far from the value 0.36 determined experimentally for a chip of mineral energy sliding on a smoothed surface prepared on a fresh fracture of the mineral.

From authors' summary

**3376. Andrews, H. I., The contact between a locomotive driving wheel and the rail,** *Wear* **2**, 6, 468-484, Oct. 1959.

In connection with improving the adhesion of locomotives, a study was made of the conditions of contact existing between locomotive driving wheels and the rail. Observations were made of such measurable quantities as contact area and shape, relative movement or "creep" in rolling, and limiting coefficient of friction, with different values of vertical load, wheel diameter, tractive force, etc. The results obtained were compared with the calculations of Hertz, Carter, and others. Whenever possible measurements were made upon actual wheels and rails, but when this was impossible the problem was simulated by a model in a material giving greater deflection under load. Consideration was also given to the effect of wear upon tires and rails. In general, reasonable agreement was obtained between actual and calculated values, except that the areas of contact could apparently be increased by roughness of the contacting surfaces, and in the case of creep there appeared to be some additional factor which has so far been neglected in calculation.

From author's summary

**3377. Denny, D. F., Time effects in the static friction of lubricated rubber,** *Wear* **2**, 4, 264-272, May 1959.

The considerable increases in static friction that occur when lubricated rubber remains at rest under load are shown to be due to squeezing out of the lubricant film from between the surfaces. Unlike the case of rigid materials, rubber continues to squeeze out the lubricant long after the surfaces have come into contact, and under certain conditions the process continues until the friction has increased to a value corresponding to unlubricated sliding.

The time-scale of the phenomenon appears to be directly related to the bulk viscosity of the lubricant and to the elastic modulus of the rubber. With rough surfaces the final friction value is greatly reduced.

From author's summary

**3378. Rubenstein, C., Review on the factors influencing the friction of fibres, yarns and fabrics,** *Wear* **2**, 4, 296-310, May 1959.

**3379. Halaunbrenner, J. B., The effect of the anisotropy of the base on rolling friction,** *Wear* **2**, 6, 423-427, Oct. 1959.

**3380. Sata, T., Transience of the state of wear by repeated rubbing,** *Wear* **3**, 2, 104-113, Mar./Apr. 1960.

It was shown many years ago that the force of friction and the rate of wear between, say, steel surfaces were reduced when one of the surfaces had first been wiped with a soft metal. Particles abraded from the rubbing surfaces have a similar effect, as author shows now by using a circular friction track, where he either allows the abraded particles to accumulate, or removes them with a cutting tool located in front of the frictional contact. In the latter case the rate of wear is much larger and remains unchanged with the distance traveled.

R. Schnurmann, England

**3381. Ogibalov, P. M., Tests on the wear of metallic parts subject to friction** (in Russian), *Inzbener. Sbornik Akad. Nauk SSSR* **28**, 190-196, 1960.

Author describes three series of experiments in which an alloy-steel disk is rubbed with a constant speed of 0.253 m/sec on a $7 \times 12 \times 60$ mm$^3$ red copper specimen under pressures of 8.1, 10.8 and 13.5 kg/mm$^2$, corresponding to loads of 1500, 2000 and 2500 kg. The disk diameter was 185.5 mm. Measurements were made after 1, 2, 3, etc. up to 20 revolutions (totalling 210) to determine the wear in mm$^3$ from loss of thickness and from loss of weight, which differ because of deformation, although in some cases the loss by weight appears to be the higher.

Keeping pressure constant, wear increases with the number of revolutions, temperature rise gradually declines. Under a load of 2500 kg the temperature of the specimen went up 233 °C in 20 revolutions.

An additional test series with loads going up by 500 kg as high as 3500 kg shows a wear in 10 revolutions of 2.5, 20.2 and 60.2 $mm^3$ at 2.7, 10.8 and 18.9 $kg/mm^2$ specific pressure.

The coefficient of friction, work done, and heat dissipation apparently were not measured.

N. B. Van Albada, Holland

**3382. Vinogradov, G. V., and Morozova, O. E., A study of the wear of steel under heavy loads with lubricants containing sulphur-based additives,** *Wear* **3**, 4, 297–308, July/Aug. 1960.

A four-ball friction machine is described. It was employed to study the wear of hardened steel at considerable contact stresses in highly purified dearomatized bright stock and solutions of sulphur and dibenzyldisulphide in the latter. Profilometric measurements were carried out, microhardnesses were determined and a metallographic study was made in the friction zone.

It is shown that estimation of the anti-wear properties of lubricants by scar diameters is only of qualitative value. The depth of the wear scars depends on the nature of the lubricant and on the sliding speed. It decreases under the action of anti-welding additives and when the sliding speed is lowered.

Under seizure conditions non etching surface layers of great hardness up to 20 or 30$\mu$ thick are formed. They probably arise in the process of rubbing as a result of diffusion to the surface of the alloy components and enrichment of the surface layers in carbon, because the hydrocarbon lubricant may act as a carburizer. The formation of very hard surface layers alleviates the heavy friction conditions at seizure.

Reaching down to a considerable depth below the nonetching layer is a layer of partially annealed steel, which forms under the action of the heat flow from the surface. The appearance of this layer facilitates plastic deformation of the steel in the friction zone and the development of adhesion between the rubbing surfaces, resulting in seizure.

The theory has been put forward that on seizure in the presence of sulphur-containing anti-welding additives submicroscopic relatively soft films of iron sulphides form first on the very hard surface layers.

The structural changes that occur in the surface layers of steel worked for a long time under heavy friction conditions in a hydrocarbon medium are different from those occurring when wear is produced in the same hydrocarbon medium in the presence of anti-welding additives.

The specific nature of heavy friction conditions is defined by the formation of a multi-layer structure in the friction zone of the steel.

From authors' summary

**3383. Schallamach, A., and Turner, D. M., The wear of slipping wheels,** *Wear* **3**, 1, 1–25, Jan. 1960.

A wheel is said to slip if its traveling velocity differs in absolute value from the circumferential velocity, or if it has a component in the direction of the axle. Slip produces sliding which is in general confined to the rear part of the area of contact, the sliding range increasing with increasing slip. Assuming proportionality between abrasion and frictional energy dissipation, wear has been calculated for a simple model as function of slip and physical properties of the wheel. At moderate slip, wear is found to increase as the square of the slip; at constant slip, wear increases with increasing stiffness, and decreases with increasing elastic hysteresis of the wheel. The results allow an estimate to be made of the dependence of wear on speed of travel under practical conditions. Experimental evidence is adduced in support of the theory.

From authors' summary

**3384. Anderson, O. L., The role of surface shear strains in the adhesion of metals: Part 1,** *Wear* **3**, 4, 253–273, July/Aug. 1960.

Author presents interesting experimental results showing "... shear strains are necessary for adhesion to occur under most circumstances when unlubricated metals are in contact...." Study of adhesion of gold ball-on-flat specimens is designed to show the importance of adhesion in friction, thus confirming aspects of work of McFarlane and Tabor [*Proc. Roy. Soc. Lond.* (*A*) **202**(1950)].

Reviewer's experience with lead specimens is that, given sufficient load (actual contact area of the order of apparent contact area) and time, adhesion will occur even without shear strains. Moreover, adhesion may or may not be the predominate factor in friction.

F. F. Ling, USA

**3385. Dearden, J., The wear of steel rails and tires in railway service,** *Wear* **3**, 1, 43–59, Jan. 1960.

The wear of rails and tires is reviewed from the point of view of the railway engineer and metallurgist, and the subject is treated on the macro- rather than the micro-scale.

After a brief statement showing the magnitude and the economics of the problem as it affects British Railways, the author gives numerical values for rail and tire wear under different conditions. The factors having an influence on wear are discussed, and some of them evaluated.

While both rails and tires have their rate of wear accelerated by atmospheric corrosion, tires have also to be re-profiled at intervals and the metal removed in this operation is often greater than that removed by wear.

Methods of reducing the wear of rails and tires are discussed.

From author's summary

**3386. Pietschmann, Elisabeth, Measuring wear of I. C. motors by means of emission spectroscopy** (in German), *Wear* **2**, 5, 335–348, Aug. 1959.

The efficiency of emission analysis for the measurement of abrasion is discussed on the basis of a survey of the copious technical literature and practical experiments. An automobile engine is used as an example for the simultaneous abrasion control of four constructional units by analyzing small lubricant samples. Compared with the modern isotope method, spectroscopic analysis combines a similar accuracy with the advantage of a simultaneous control of several constructional units without the necessity for special precautionary measures; the discontinuous sampling, however, may constitute a certain drawback of the spectroscopic method.

From author's summary

**3387. Lees, H. D., Statistics on cylinder wear in marine diesel engines,** *Wear* **2**, 4, 273–295, May 1959.

Paper describes the United Steamship Company's efforts to elucidate statistically the problems in connection with cylinder wear in marine diesel engines, by the use of punched cards and electronic digital computers. Further, some hypotheses concerning the causes of cylinder wear and the way to overcome it are given.

From author's summary

**3388. Tanner, R. I., Non-Newtonian flow and the oil seal problem,** *J. Mech. Engng. Sci., Lond.* **2**, 1, 25–28, Mar. 1960.

The intriguing question of how oil films are formed in rotary shaft seals, evidently separating the sealing surfaces without allowing leakage if the load is high enough, is discussed in particular reference to possible non-Newtonian effects. Jagger [*Proc. Inst. Mech. Engrs.* **171**, p. 597, 1957; see also AMR **10**(1957), Rev. 3501] demonstrated experimentally the existence of such films and proposed that an oil-air meniscus formed by surface tension at the outside edge of the interface prevented leakage. Everything from the Weissenberg effect to the generation of elastic ripples in the contacting surfaces to support the load and to prevent leakage was proposed in the lively discussion to Jagger's paper. As a sequel

to this discussion, the author presents a general statement in tensor notation of the non-Newtonian flow of a lubricant in which stresses transverse to the direction of flow arise naturally without the postulation of so-called "cross-viscosities," By applying this theory to the case of a face-type shaft seal (parallel-surface annular rings) and by accepting the conclusions of Oldroyd [AMR 12(1959), Rev. 1331] on the non-Newtonian properties of fluids (based on data for polymer solutions), he concludes that non-Newtonian effects are not likely to provide a complete explanation of the oil-seal problem. The author states that, under these conditions, the effective cross-flow viscosity always remains finite and no normal film pressures greater than ambient are generated. He proposes (a) partial "dry" contact, (b) elastic deformation causing nonplane seal faces, and (c) surface tension, in combination with non-Newtonian effects, as possible explanations of the observed results, but defers further analysis until more experimental evidence is obtained.

The reviewer notes that the sealing ability and load-carrying capacity of seals may arise from thermal distortions and extraneous vibrations of the sealing elements without recourse to non-Newtonian flow. Dayton ["Mechanical Wear," ASM (1950), pp. 165-180] proposed that wedge-shaped films are formed by thermal expansion of the surfaces resulting from the heat generated by viscous shearing of the oil film. The load-carrying effect of vibrations in conjunction with cavitation in the oil film was calculated by Nahavandi and Osterle [ASME Paper no. 60-LUBS-3 (March 1960)]. Assuming that the fluid is compressible, Taylor and Saffman [AMR 11(1958), Rev. 708] demonstrated both normal pressures and a radial pressure gradient between a rotating disk face and a nonrotating plane that were subjected to vibration.

Although the reviewer did not check all the derivations presented, the theory in this paper should provide the basis for analysis of non-Newtonian effects in lubrication problems.

L. B. Sibley, USA

**3389. Sasaki, T., Mori, H., and Okino, N., Theory of grease lubrication of cylindrical roller bearing,** *Bull. JSME* **3**, 10, 212-219, May 1960.

This paper derives the pressure, velocity, and film thickness found in a "Bingham Plastic" lubricant between rotating cylinders. The following assumptions are made:

(1) Two-dimensional flow
(2) One-dimensional pressure distribution
(3) Perfectly rigid rollers
(4) Lubricant follows the law $\tau = I\tau_0 + y$ (shear rate), ($\tau$ is shear stress in lubricant, $\tau_0$ is a yield shear stress, $y$ is a plastic viscosity coefficient)
(5) Lubricant incompressible
(6) Plastic viscosity constant with temperature and pressure
(7) No viscoelasticity
(8) No inertia effect

The formation of a "core" of zero shear ($\tau < \tau_0$) is investigated. Two special cases are considered, and the theory is applied to roller bearings. One effect of the plastic yield stress is to introduce starting torque into the theory. In general, as velocities increase, behavior approaches that predicted by ordinary hydrodynamic theory.

E. P. Kingsbury, USA

**3390. Osterle, J. F., Film geometry effects in hydrodynamic gear lubrication,** *Wear* **2**, 6, 416-422, Oct. 1959.

The involute gear lubrication problem is analyzed with a first-order correction made to the conventional parabolic arc film thickness equation to account for the actual tooth shape and the squeeze action resulting from its time dependence. Both internal and external gears are considered for one pressure angle and a range of gear ratios. The film geometry effect is found to be greater for internal than for external gears and to decrease as the minimum film thickness decreases. Conclusions are drawn concerning the importance of the correction.

From author's summary

**3391. Lancaster, P. R., and Rowe, G. W., A comparison of boundary lubricants under light and heavy loads,** *Wear* **2**, 6, 428-437, Oct. 1959.

This paper describes an experimental study of the lubrication of steel bars during slow drawing under very heavy loads (up to 10 tons), using dies of high entry-angle to discourage hydrodynamic contributions. The results are compared with those obtained using the same materials at light loads (1-4 kg). Soap and fatty acids are used to represent good boundary lubricants, while silicone oil is used as a standard poor lubricant and as a carrier for surface-active additives.

When the loads are increased from a few kilograms to a few tons the relative behavior of the lubricants is the same, provided that the surface is not stretched more than a few per cent. If the loads are further increased the interfacial pressure (20-30 t.p.s.i..) is not greatly altered but the surface extends and the lubricant film breaks down, resulting in metallic pickup on the die. This can, however, be avoided by providing a regular pattern of pockets on the surface to trap the lubricant. A convenient way of doing this is to grit-blast the surfaces. When a reserve supply of lubricant is provided in this way in the vital region between die and specimen the lubrication does not break down. Under these conditions the boundary lubricating characteristics are again an important factor in the lubrication. The cross-sectional area of the bar may be reduced by up to nearly 60% in a single pass using soap as lubricant on a grit-blasted surface without pickup occurring. The limit is set not by pickup but by the strength of the drawn bar.

From authors' summary

**3392. Hirano, F., and Yamamoto, S., Four-ball test on lubricating oils containing solid particles,** *Wear* **2**, 5, 349-363, Aug. 1959.

In order to investigate the effect of solid particles in lubricating oils, systematic research was carried out using a four-ball machine. The effects of hardness, concentration of particles and viscosity of oil were specially taken into consideration. Softer particles like metal powder do not penetrate into the contact surfaces, but accumulate at the front of the contact area and disturb the formation of an oil film, causing a slight amount of abrasion. On the other hand, particles such as quartz powder or carborundum are easily introduced into the contact area, causing marked increase in abrasion. There is a straightforward relationship between the area of impression and the concentration of particles. The test with the Timken machine gives similar results.

From authors' summary

**3393. Constantinescu, V. N., Unsteady lubrication in a turbulent region** (in French), *Rev. Mécan. Appl.* **4**, 1, 73-96, 1959.

Using some ideas and results given previously [AMR 12(1959), Revs. 4258, 4259], author considers further the turbulent flow within lubricant film under nonsteady conditions.

G. Power, England

**3394. Smith, F. W., Lubricant behaviour in concentrated contact systems—the castor oil-steel system,** *Wear* **2**, 4, 250-263, May 1959.

Experiments are described in which frictional forces are measured between hardened steel surfaces lubricated with castor oil. In one series of experiments, balls of different sizes slide against a moving plane at a maximum Hertz stress of 389,000 psi (273.5 $kg/mm^2$) over a range of speeds up to 140 cm/sec. In another experiment the frictional force is measured between rollers which roll with a small component of sliding motion, the elastic stresses being similar to those used in the first experiment.

In an interpretation of these measurements, it is concluded that in a certain range of speed most of the frictional force is transmitted through a film of lubricant acting as a plastic solid whose shear strength decreases with increasing temperature, and that in the sliding experiment the coefficient of friction gives essentially a measure of the temperature rise in the lubricant.

It is further suggested that in concentrated contact systems of this type where the frictional force is determined by the high-stress plastic yield of the lubricant, the film thickness is determined by viscous flow properties.

From author's summary

**3395. Rozeanu, L., and Preotescu, O., Specification of the viscosity of lubricants,** *Wear* **2,** 6, 456–467, Oct. 1959.

**3396. Heath, H. H., and Phillips, K. F., An investigation into sliding bearings suitable for a $CO_2$-cooled nuclear reactor,** *Wear* **3,** 5, 358–373, Sept./Oct. 1960.

In a nuclear reactor a number of mechanisms are required to operate up to about 400 °C, often in the presence of intense radiation fluxes. Three possible solutions to the problem of making bearings for these mechanisms are discussed; the use of unlubricated metals, of massive graphite, and of thin graphite films on a metal base.

Three metals, two nitrided steels and a tool steel, have been found to run well unlubricated, both against themselves and a variety of other steels. Coefficients of friction for unlubricated metals are high, usually in the range 0.3 to 0.8 but tending on occasions towards unity. An electrographite was found which would run satisfactorily in dry $CO_2$ but the performance in dry air, dry nitrogen and in a vacuum was poor. Thin graphite films have been found to have too unpredictable a life for practical purposes.

From authors' summary

**3397. Elwell, R. C., and Sternlicht, B., Theoretical and experimental analysis of hydrostatic thrust bearings,** *ASME Trans.* **82** D *(J. Basic Engng.)*, 3, 505–512, Sept. 1960.

Theoretical and experimental analysis of two types of circular hydrostatic thrust bearings are presented together with design equations for orifice, capillary and constant flow restriction. Good agreement is obtained between theory and experiment. Each type of restriction imparts its own characteristics on the oil flow, load capacity and stiffness. Thus design analysis requires consideration of the bearing system as a whole.

C. F. Kettleborough, Australia

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